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ABSTRACT

This course is designed to review the arithmetic skills used by many Marines in the daily pursuance of their duties. It consists of six study units: (1) number systems and operations; (2) fractions and percents; (3) introduction to algebra; (4) units of measurement (considering both the metric and United States systems); (5) geometric forms; and (6) problem solving. Each unit begins with a general objective and is divided into numbered work units, each presenting one or more specific objectives. Motivation and interest in each topic is developed by use of practical problems, and the presentation of the history and principles underlying each subject. Exercises (with answers) are included throughout the units and the manual concludes with a review lesson designed to better prepare students for the final examination and consisting of questions and an answer sheet to be filled out and returned to the Marine Corps Institute. Supporting information (such as materials needed and forms for lesson submission, grading, and assistance) is also included. (JN)

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13.34g
21 Dec 84

1. ORIGIN

MCI course 13.34g, Math for Marines, has been prepared by the Marine Corps Institute.

2. APPLICABILITY

This course is for instructional purposes only.

J. M. D. HOLLADAY
Lieutenant Colonel, U. S. Marine Corps
Deputy Director

INFORMATION

FOR

MCI STUDENTS

Welcome to the Marine Corps Institute training program. Your interest in self-improvement and increased professional competence is commendable.

Information is provided below to assist you in completing the course. Please read this guidance before proceeding with your studies.

1. MATERIALS

Check your course materials. You should have all the materials listed in the "Course Introduction." In addition you should have an envelope to mail your review lesson back to MCI for grading unless your review lesson answer sheet is of the self-mailing type. If your answer sheet is the pre-printed type, check to see that your name, rank, and social security number are correct. Check closely, your MCI records are kept on a computer and any discrepancy in the above information may cause your subsequent activity to go unrecorded. You may correct the information directly on the answer sheet. If you did not receive all your materials, notify your training NCO. If you are not attached to a Marine Corps unit, request them through the Hotline (autovon 288-4175 or commercial 202-433-4175).

2. LESSON SUBMISSION

The self-graded exercises contained in your course are not to be returned to MCI. Only the completed review lesson answer sheet should be mailed to MCI. The answer sheet is to be completed and mailed only after you have finished all of the study units in the course booklet. The review lesson has been designed to prepare you for the final examination.

It is important that you provide the required information at the bottom of your review lesson answer sheet if it does not have your name and address printed on it. In courses in which the work is submitted on blank paper or printed forms, identify each sheet in the following manner:

DOE, John J. Sgt 332-11-9999
08.4g, Forward Observation
Review Lesson
Military or office address
(RUC number, if available)

Submit your review lesson on the answer sheet and/or forms provided. Complete all blocks and follow the directions on the answer sheet for mailing. Otherwise, your answer sheet may be delayed or lost. If you have to interrupt your studies for any reason and find that you cannot complete your course in one year, you may request a single six month extension by contacting your training NCO, at least one month prior to your course completion deadline date. If you are not attached to a Marine Corps unit you may make this request by letter. Your commanding officer is notified monthly of your status through the monthly Unit Activity Report. In the event of difficulty, contact your training NCO or MCI immediately.

3. MAIL-TIME DELAY

Presented below are the mail-time delays that you may experience between the mailing of your review lesson and its return to you.

	TURNAROUND MAIL TIME	MCI PROCESSING TIME	TOTAL NUMBER DAYS
EAST COAST	16	5	21
WEST COAST	16	5	21
FPO NEW YORK	18	5	23
FPO SAN FRANCISCO	22	5	27

You may also experience a short delay in receiving your final examination due to administrative screening required at MCI.

4. GRADING SYSTEM

LESSONS			EXAMS	
GRADE	PERCENT	MEANING	GRADE	PERCENT
A	94-100	EXCELLENT	A	94-100
B	86-93	ABOVE AVERAGE	B	86-93
C	78-85	AVERAGE	C	78-85
D	70-77	BELOW AVERAGE	D	65-77
NL	BELOW 70	FAILING	F	BELOW 65

You will receive a percentage grade for your review lesson and for the final examination. A review lesson which receives a score below 70 is given a grade of NL (no lesson). It must be resubmitted and PASSED before you will receive an examination. The grade attained on the final exam is your course grade, unless you fail your first exam. Those who fail their first exam will be sent an alternate exam in which the highest grade possible is 65%. Failure of the alternate will result in failure of the course.

5. FINAL EXAMINATION

ACTIVE DUTY PERSONNEL: When you pass your REVIEW LESSON, your examination will be mailed automatically to your commanding officer. The administration of MCI final examinations must be supervised by a commissioned or warrant officer or a staff NCO.

OTHER PERSONNEL: Your examination may be administered and supervised by your supervisor.

6. COMPLETION CERTIFICATE

The completion certificate will be mailed to your commanding officer and your official records will be updated automatically. For non Marines, your completion certificate is mailed to your supervisor.

7. RESERVE RETIREMENT CREDITS

Reserve retirement credits are awarded to inactive duty personnel only. Credits awarded for each course are listed in the "Course Introduction." Credits are only awarded upon successful completion of the course. Reserve retirement credits are not awarded for MCI study performed during drill periods if credits are also awarded for drill attendance.

8. DISENROLLMENT

Only your commanding officer can request your disenrollment from an MCI course. However, an automatic disenrollment occurs if the course is not completed (including the final exam) by the time you reach the CCD (course completion deadline) or the ACCD (adjusted course completion deadline) date. This action will adversely affect the unit's completion rate.

9. ASSISTANCE

Consult your training NCO if you have questions concerning course content. Should he/she be unable to assist you, MCI is ready to help you whenever you need it. Please use the Student Course Content Assistance Request Form (ISD-1) attached to the end of your course booklet or call one of the AUTOVON telephone numbers listed below for the appropriate course writer section.

PERSONNEL/ADMINISTRATION	288-3259
COMMUNICATIONS/ELECTRONICS/AVIATION	
NBC/INTELLIGENCE	288-3604
INFANTRY	288-3611
ENGINEER/MOTOR TRANSPORT	288-2275
SUPPLY/FOOD SERVICES/FISCAL	288-2285
TANKS/ARTILLERY/INFANTRY WEAPONS REPAIR	
LOGISTICS/EMBARKATION/MAINTENANCE MANAGEMENT/	
ASSAULT AMPHIBIAN VEHICLES	288-2290

For administrative problems use the OAR or call the MCI HOTLINE: 288-4175.

For commercial phone lines, use area code 202 and prefix 433 instead of 288.

PREFACE

Math for Marine supplies a much needed review of the arithmetic skills used by many Marines in the daily pursuance of their duties. Although arithmetic is stressed, the course covers topics such as solving simple algebraic equations, finding the area and volume of basic geometric forms, and use of the Pythagorean relationship. Motivation and interest in each topic is developed by use of practical problems, and the presentation of the history and principles underlying each subject. Exercises are presented throughout the course to better prepare the student for the submitted written assignment and the examination.

SOURCE MATERIALS

Mathematics Workbook, Marine Corps Engineer School, 1965.
Percentage. Navy Air Technical Training Command, 1965.
Naval Correspondence Course Center, Mathematics Part I 1976.
Naval Correspondence Course Center, Mathematics Part II 1975.
Naval Correspondence Course Center, Mathematics Part III 1971.

MATH FOR MARINES

Course Introduction

MATH FOR MARINES is designed as a review of basic mathematics for anyone with at least a junior high school background.

ADMINISTRATIVE INFORMATION

ORDER OF STUDIES

Study Unit Number	Study Hours	Subject Matter
1	2	Number Systems and Operations
2	3	Fractions and Percents
3	3	Introduction to Algebra
4	3	Units of Measurement
5	3	Geometric Forms
6	2	Problem Solving
	3	Review Lesson
	3	FINAL EXAMINATION
	22	

RESERVE RETIREMENT CREDITS: 7

EXAMINATION: Supervised final examination without textbook or notes with a time limit of 3 hours.

MATERIALS: MCI 13.34g, Math for Marines
Protractor
Review lesson and answer sheet

RETURN OF MATERIALS: Students who successfully complete this course are permitted to keep the course materials.

Students disenrolled for inactivity or at the request of their commanding officer will return all course materials.

HOW TO TAKE THIS COURSE

This course contains 6 study units. Each study unit begins with a general objective which is a statement of what you should learn from that study unit. The study units are divided into numbered work units, each presenting one or more specific objectives. Read the objective(s) and then the work unit text. At the end of the work unit text are study questions which you should be able to answer without referring to the text of the work unit. After answering the questions, check your answers against the correct ones listed at the end of the study unit. If you miss any of the questions, you should restudy the text of the work unit until you understand the correct response. When you have mastered one study unit, move on to the next. After you have completed all study units, complete the review lesson and take it to your training officer or NCO for mailing to MCI. MCI will mail the final examination to your training officer or NCO when you pass the review lesson.

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MARINE CORPS INSTITUTE

STUDY GUIDE

Welcome to the Marine Corps Institute correspondence training program. By enrolling in this course, you have shown a desire to improve the skills you need for effective job performance, and MCI has provided materials to help you achieve your goal. Now all you need is to develop your own method for using these materials to best advantage.

The following guidelines present a four-part approach to completing your MCI course successfully:

1. Make a "reconnaissance" of your materials;
2. Plan your study time and choose a good study environment;
3. Study thoroughly and systematically;
4. Prepare for the final exam.

I. MAKE A "RECONNAISSANCE" OF YOUR MATERIALS

Begin with a look at the course introduction page. Read the COURSE INTRODUCTION to get the "big picture" of the course. Then read the MATERIALS section near the bottom of the page to find out which text(s) and study aids you should have received with the course. If any of the listed materials are missing, see Information for MCI Students to find out how to get them. If you have everything that is listed, you are ready to "reconnoiter" your MCI course.

Read through the table(s) of contents of your text(s). Note the various subjects covered in the course and the order in which they are taught. Leaf through the text(s) and look at the illustrations. Read a few work unit questions to get an idea of the types that are asked. If MCI provides other study aids, such as a slide rule or a plotting board, familiarize yourself with them. Now, get down to specifics!

II. PLAN YOUR STUDY TIME AND CHOOSE A GOOD STUDY ENVIRONMENT

From looking over the course materials, you should have some idea of how much study you will need to complete this course. But "some idea" is not enough. You need to work up a personal study plan; the following steps should give you some help.

(A) Get a calendar and mark those days of the week when you have time free for study. Two study periods per week, each lasting 1 to 3 hours, are suggested for completing the minimum two study units required each month by MCI. Of course, work and other schedules are not the same for everyone. The important thing is that you schedule a regular time for study on the same days of each week.

(B) Read the course introduction page again. The section marked ORDER OF STUDIES tells you the number of study units in the course and the approximate number of study hours you will need to complete each study unit. Plug these study hours into your schedule. For example, if you set aside two 2-hour study periods each week and the ORDER OF STUDIES estimates 7 study hours for your first study unit, you could easily schedule and complete the first study unit in one study period. On your calendar you would mark "Study Unit 1" on the

appropriate day. Suppose that the second study unit of your course requires 3 study hours. In that case, you would divide the study unit in half and work on each half during a separate study period. You would mark your calendar accordingly. Indicate on your calendar exactly when you plan to work on each study unit for the entire course. Do not forget to schedule one or two study periods to prepare for the final exam.

(C) Stick to your schedule.

Besides planning your study time, you should also choose a study environment that is right for you. Most people need a quiet place for study, like a library or a reading lounge; other people study better where there is background music; still others prefer to study out-of-doors. You must choose your study environment carefully so that it fits your individual needs.

III. STUDY THOROUGHLY AND SYSTEMATICALLY

Armed with a workable schedule and situated in a good study environment you are now ready to attack your course study unit by study unit. To begin, turn to the first page of study unit 1. On this page you will find the study unit objective, a statement of what you should be able to do after completing the study unit.

DO NOT begin by reading the work unit questions and flipping through the text for answers. If you do so, you will prepare to fail, not pass, the final exam. Instead, proceed as follows:

(A) Read the objective for the first work unit and then read the work unit text carefully. Make notes on the ideas you feel are important.

(B) Without referring to the text, answer the questions at the end of the work unit.

(C) Check your answers against the correct ones listed at the end of the study unit.

(D) If you miss any of the questions, restudy the work unit until you understand the correct response.

(E) Go on to the next work unit and repeat steps (A) through (D) until you have completed all the work units in the study unit.

Follow the same procedure for each study unit of the course. If you have problems with the text or work unit questions that you cannot solve on your own, ask your section OIC or NCOIC for help. If he cannot aid you, request assistance from MCI on the Student Course Content Assistance Request included with this course.

When you have finished all the study units, complete the course review lesson. Try to answer each question without the aid of reference materials. However, if you do not know an answer, look it up. When you have finished the lesson, take it to your training officer or NCO for mailing to MCI. MCI will grade it and send you a feedback sheet listing course references for any questions that you miss.

IV. PREPARE FOR THE FINAL EXAM

How do you prepare for the final exam? Follow these four steps:

(A) Review each study unit objective as a summary of what was taught in the course.

(B) Reread all portions of the text that you found particularly difficult.

(C) Review all the work unit questions, paying special attention to those you missed the first time around.

(D) Study the course review lesson, paying particular attention to the questions you missed.

If you follow these simple steps, you should do well on the final. GOOD LUCK!

STUDY UNIT 1

NUMBER SYSTEMS AND OPERATIONS

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO IDENTIFY THE COMPONENTS OF THE REAL NUMBER SYSTEM AND THE BASIC PROPERTIES OF NUMBERS. YOU WILL RECOGNIZE THE SYMBOLS OF GROUPING AND BE ABLE TO SIMPLIFY EXPRESSIONS THAT USE THEM.

Every Marine, from a scout in a fireteam to the Commandant, uses numbers in some form every day. Numbers are an inescapable item in our modern way of life: Marine Corps strength reduced to 190,000. The choppers will lift 56 people from "L" Company 13 miles to objective Charlie. Rations and ammo are needed for the company for 3 days in the field. How many cubic yards of dirt must be removed from the hill in order for the road to pass? How many 5-ton dump truck loads is this? How much fuel will the tank platoon need to travel 127 miles? Check the map and see how far it is to the spot where the enemy were spotted. The PX is having a 25% discount sale. PFC EXPERT fired 237 with the M16. Compute Pvt U. A. BRIGTIME's new PEBD. These are just a few of the everyday situations involving numbers. Where numbers are involved, there is a need to know how to work with them. Usually it will be a simple matter of arithmetic done without a second thought, but once in a while something is needed where we must rely on the memory of some long forgotten item from Hinky High School. Although arithmetic is the thing most often needed, some Marines need to know a little more for work in surveying, data processing, electronics, and other technical subjects. Even these Marines though must have a good grounding in arithmetic. This course will present some basic ideas about numbers and arithmetic. The student will be taken up to an introduction to algebra and the simple equation. For most people it should be a matter of review of the fundamental concepts of arithmetic.

Do you remember how you originally learned or did not learn these fundamentals? For example, how did you learn to divide fractions? More than likely the teacher said, "Today we are going to divide fractions. This is done by inverting the divisor and multiplying." She then showed an example on the board and told you to turn to page 28 and do problems 1-25. You may have learned to divide fractions, but did you ever wonder WHY you inverted that divisor? This is only one of the many things in arithmetic about which you never learned WHY. This is the reason for the "fear" of mathematics by so many people. They don't know why they are doing a particular operation, consequently they don't understand what they are doing and learning comes to a standstill because there is no basis for further progress in the subject.

Work Unit 1-1. HISTORY OF THE REAL NUMBERS

IDENTIFY THE SINGLE EVENT THAT MADE THE HINDU-ARABIC NUMBER SYSTEM SUPERIOR TO ALL OTHERS.

DETERMINE THAT THE DIGITS IN ANY NUMBER HAVE TWO PURPOSES.

In a primitive society man had very little, if any, use for numbers. There was no need for inventorying 782 gear or counting money (there wasn't any). There were no clicks right or left, or up or down with the M1 slingshot or the M1A1 spear. The hunting party didn't sweat counting "C" rats or water cans. Charlie Caveman carried his only possession, his weapon, and moved to where the hunting was good. When Charlie's wife said, "Enough of this, I want a cave of my own," they stopped the nomadic life and took up residence in Cavetown where they naturally started to accumulate possessions, one of the first steps of the civilizing process. This settling down and cessation of wandering led to farming, and with farming came the keeping of herds and flocks of animals. At this stage numbers were still not a necessity. In fact, anthropologists measure the degree of civilization by the number of number words that a culture has. Many primitive tribes today use only sounds to indicate one, two, and many. If you own two or three sheep, a visual inspection of the pen is all that is needed to account for them. When this flock or herd amounts to 10 or 15 animals, however, a glance is not enough to tell if one is missing. Charlie designed several ingenious (for him) ways of keeping track of his herd. His method was actually one that is basic to our present-day mathematics. It is based on the principle of one-to-one correspondence. He simply matched the number of animals with items in a collection of objects.

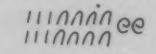
For example, maybe as he let the animals out in the morning he tied a knot in a rope for each animal. When they came back in the evening he matched each knot with an animal. If he had knots left over, he had animals missing. He did the same thing by piling up pebbles or sticks, each one representing an animal. As Charlie became more rational, he tired of those processes for accounting for his possessions. His next step toward the beginning of numbers was the formation of model groups. This was just an extension of his one-to-one correspondence or matching process. It involved using well-known items from his environment to symbolize a given collection of objects. Each symbol had a characteristic that was typical of the model group. For example, the wings of a bird symbolized two; the leaves of a clover, three; the legs of a sheep, four; the fingers on a hand, five; the petals on a particular flower, six; and so on. In his mind, Charlie replaced that pile of five stones with the number of fingers on his hand. If he had eight fishhooks he matched them up with the model group that he used to signify eight. Naturally, the evolution brought about using the name for the model group instead of the actual symbol. Although it still was not counting, the model group was a major step toward it. It simply provided a recognition of the cardinal (how many) value of number. Later, when man started to systematize his model groups and organize them in sequence (each succeeding the other by one), he was on his way to counting.

Anthropologists have shown us that all of the early civilized cultures had number systems and counting. The Egyptians used a system that started with tally marks which were continued until 10 which was replaced by the symbol for a heelbone. Each succeeding power of 10 was replaced by a different symbol. The system was based on addition and repetition. That is, add up the repeated symbols (fig 1-1).

NUMBER	SYMBOL
1	Tally mark
10	Heel bone
100	Scroll
1,000	Lotus flower
10,000	Bent line
100,000	Burbot
1,000,000	Man in astonishment

Examples of use:

1502

286 

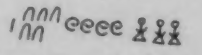
3451 

Fig 1-1. Symbols for Egyptian numbers.

Most everyone is familiar with the Roman system of notation because it is still taught in most schools and it is still used to some extent for decoration on monuments and buildings, and for dates. Basically this system is also founded on the principles of addition and repetition although there are several other wrinkles added. The Roman system greatly reduced the symbolism that was required in the Egyptian system, but it still did not approximate the value of the Hindu-Arabic number system which came into being possibly as early as 500 or 600 BC. The nomadic Arabian tribes picked up the Hindu notation in their travels and carried its use to Europe. It was several hundred years, however, before it replaced the Roman system.

The Hindu-Arabic system differs from the Egyptian and Roman systems in that a different symbol is used for each number. It is similar to the others systems in that it is based on addition, meaning that any number's value is a sum of its parts. It is similar to the Egyptian system in that there is a compounding at each power of ten. This simply means that the Hindu-Arabic system is a decimal system. An important similarity to the Roman system is the position of the numerals. Moving the symbols within the number changes the value of the number. The unique principles of the Hindu-Arabic system was the idea of place value which no system had previously employed. In its early use, the Hindu-Arabic system had no distinct advantage over the number system of any other culture. The single event that made the Hindu-Arabic number system superior to all others was the invention of zero sometime in the 9th century AD. The zero serves as a place holder and a signifier of "no value," or "nothing." It is when used in this context with the principle of place value that the zero takes on such great significance. As you know, the Hindu-Arabic system uses 10 basic symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Each of these digits has a cardinal value, that is, a "how many" value, and represents a model group. Whereas three knots in a rope signified the idea of three, now the symbol 3 indicates "threeness," and so on for the other symbols. In any number each of the numerals or digits signifying it has two purposes; one is to indicate the cardinal value just mentioned and the other is to show how many of what, or the place value. That is, how many units, tens of units, hundreds of units, thousands of units, and so on.

With the aid of the principle of addition, compounding of units becomes possible. To show the next number after nine we write 10 which symbolizes one ten and no units. Here the zero is functioning as a placeholder occupying space in the units column. It shows that there are no units involved in this number. The next number is 11 which is one ten and one unit. The next number is 12 which is one ten and two units. This continues until 99 which is nine tens and nine units after which we must use three digits. The numerals 100 signify one hundred, no tens, and no units. The numerals 463 signify four hundreds, six tens, and three units. The numerals 999 signify nine hundreds, nine tens, and nine units. After this, four digits are needed. You should see from this the validity of the statement that the system is unlimited in size.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. What single factor made the Hindu-Arabic number system so superior?

   a. Based on addition
   b. Compounding of powers of ten
   c. Use of zero
   d. Repetition

2. The symbol in the Hindu-Arabic number system which serves as a place holder and shows no value is the _____.

3. The purpose of the digits in any number is to show the cardinal value and the _____ value.

4. Which statement represents a cardinal number?

   a. Building number 160
   b. Telephone number OX8-3604
   c. John Unitas wears number 19
   d. PFC HERO took 10 grenades

Work Unit 1-2. THE NATURAL NUMBERS

IDENTIFY THE NATURAL NUMBERS IN THE COUNTING PROCESS.

IDENTIFY THE IMPORTANT PRINCIPLE OF CLOSURE.

Remember that when Charlie Caveman put the model groups into an organized sequence, this was the forerunner to counting. The successive model groups in his counting system each increased by one the size of the group that preceded it. This was the key to the counting process (increase by one). Now man had the means to count, and the numbers he used to do this counting

are called the natural numbers. They are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . (The three dots in mathematics indicate and so on). Note that the counting or natural numbers do not include zero (fig 1-2).

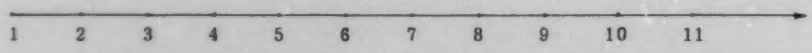


Fig 1-2. Graphical representation of the natural numbers.

In order to understand the difference between the various systems and why they developed, it is necessary to talk about one important principle of number called closure. Other number principles will be discussed in a later work unit, but closure must be discussed here in order to progress to the other systems. If you add any two natural numbers, will the sum be a natural number? A little thought and a couple of experimental problems should produce the answer, yes. For example, if you add 9 and 8, 10 and 27, 99 and 2369, 9872 and 10463, or any two natural numbers, you should see that the result will always stay within the confines of the system. When this occurs, the system is said to be closed under addition. How about multiplication? Will the product of any two natural numbers always be a natural number? Since multiplication is just a form of repeated additions, a quick review of the system should show you that the natural numbers are closed under multiplication. That is, the product of any two natural numbers will always be a natural number. How about subtraction? Is the difference between any two natural numbers a natural number? Again look over the system and try a few examples. It should not take you too long to see that problems such as 7 - 10 and 20 - 25 cannot be answered with a natural number. The natural numbers are not closed under subtraction. One example, $\frac{4}{3} = ?$, is enough to show that the natural numbers are not closed under division.

What does all of this mean to us? What is the importance of closure? The early cultures had extremely little use for numbers. Counting, mainly to keep track of possessions, was their primary use. Charlie Caveman had no need for anything other than the natural numbers, but with civilized cultures came the development of towns and the use of money which you know, if you have ever operated a budget, places a premium on knowledge of numbers. Closure shows us that the natural numbers are not sufficient to handle all of the operations needed. There was a need for other number systems.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. The key to the counting process with the natural numbers was to increase by _____.

2. Which set consists entirely of natural numbers?

   a. 3, 17, 19.5
   b. 0, 1, 2
   c. 3, 15, 27
   d. 1/2, 2, 10

3. The principle of closure states that when you add two natural numbers, the sum will always be a _____ number.

4. Which of the below illustrates the principle of closure?

   a. 4 - 9 =
   b. 4 + 9 =
   c. 4 ÷ 9 =
   d. 9 ÷ 4 =

Work Unit 1-3. THE INTEGERS

IDENTIFY THAT INTEGERS ARE A SYSTEM OF WHOLE NUMBERS COMPRISED OF TWO NUMBER SYSTEMS.

The integers are the system of whole numbers. Of course the natural numbers are whole numbers, but not the complete set that man was to need. With the use of subtraction there came a need for an expanded system. Recall that it was mentioned that problems such as 7 - 10 and 20 - 25 could not be answered with the natural numbers. The answers to both of these problems require a system that includes negative numbers (fig 1-3). Note that the integers start at and include zero, and extend indefinitely in both directions from this point.

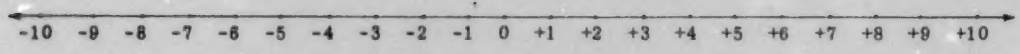


Fig 1-3. Graphical representation of the integers.

If you perform several checks on this system you will see that it is closed under addition, multiplication, and subtraction, but again a simple problem such as $\frac{3}{2}$ = ? is enough to show that the integers are not closed under division.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. The answer to which of the below listed problems is an integer?

   a. 3/2
   b. 12.6 - 4
   c. 8 - 12
   d. 7 1/4 x 5

2. Integers are a system of whole numbers made up of both natural and _____ numbers.

Work Unit 1-4. THE RATIONAL NUMBERS

DETERMINE THAT RATIONAL NUMBERS ARE NUMBERS THAT CAN BE EXPRESSED AS FRACTIONS.

Rational numbers are simply numbers that can be expressed as fractions, that is, the quotient of two integers. Clearly man must have had a need for the rational numbers before the integers, but the integers were discussed first because of their association with subtraction which man probably used very early in his number career. The rational numbers include the integers (5, 6, 7), terminating and repeating decimal fractions (.25 = $\frac{1}{4}$, .333... = $\frac{1}{3}$), and all fractional numbers (fig 1-4). It should be evident that the system is closed under all four of the basic operations: addition, subtraction, multiplication, and division. That is, any of these operations performed on a number in the rational number system will give us a rational number as the answer. Again note that the integers are part of the rational numbers.

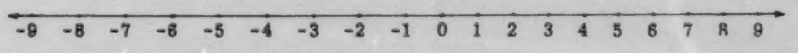


Fig 1-4. Graphical representation of the rational numbers.

The three number systems that we have just mentioned (natural, integers, rational) and one additional system (irrational numbers) make up the real number system which is the central number system of mathematics (fig 1-5). The graphical representation of the real numbers would look the same as the rational numbers, and for all practical purposes for our use they are the same. The irrational numbers are numbers such as $\sqrt{2}$, π, and non-terminating, non-repeating decimals. They will not be discussed here.

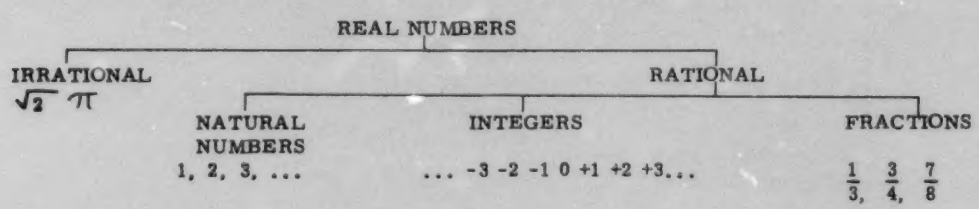


Fig 1-5. The real number system.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. Rational numbers are numbers that can be expressed as _____.

2. The mathematical expression .75 = 3/4 is a _____ number.

Work Unit 1-5. PROPERTIES OF NUMBERS

IDENTIFY THE THREE AXIOMS OF EQUALITY

In the pre-1950 era of mathematical instruction, little was ever taught about why we can do some of the things that we do with numbers. Now we are teaching the why in addition to the how. Upon analysis you will see that much of this why is self-evident. This is actually what an axiom is, a self-evident truth. Because you have been taught how to perform these operations and you now do them without second thought, the introduction of the properties may seem at first irrelevant and unnecessary at this point since they are so obvious. But this is judgment based on hindsight. Place yourself in the position of the beginner in the study of arithmetic and imagine the value that could be gained by knowing the fundamental properties of a number. Even at this stage of your mathematical education, an understanding of these properties can lead to an increased ability to handle more complicated problems.

Axioms of equality. There are three axioms of equality. (An equality is simply a statement that two things are equal: a = b.) They will be particularly important in the study of equations, although they also have important applications in your work in arithmetic.

The reflexive property of equality. This is sometimes referred to as the law of identity. It is simply a long-winded way of saying that any number reflects itself or is equal to itself. Stated symbolically: For any number x, x = x.

The symmetric property of equality. This states that an equality is reversible. Stated symbolically: For any numbers x and y, if x = y, then y = x.

The transitive property of equality. This makes it possible to identify two numbers as being equal if each of them is equal to a third quantity. Stated symbolically: For any numbers x, y, and z, if x = y and y = z, then x = z.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. When you write a statement such as a = 6, what axiom of equality allows you to change this to 6 = a?

   a. Commutative property
   b. Symmetric property
   c. Reflexive property
   d. Associative property

2. What axiom of equality applies to this statement? If a = b and b = c, then a = c.

   a. Transitive property
   b. Commutative property
   c. Reflexive property
   d. Distributive property

3. Which item illustrates the reflexive property of equality?

   a. 26 = a
   b. a = b
   c. a = a
   d. b = 26

IDENTIFY THE COMMUTATIVE PROPERTY WITH RESPECT TO ADDITION AND MULTIPLICATION.

IDENTIFY THE ASSOCIATIVE PROPERTY WITH RESPECT TO ADDITION AND MULTIPLICATION.

Commutative and associative properties. There are certain laws that are obeyed by various number systems. In fact, number systems can be categorized by their conformity to these laws. Two of the basic laws are the commutative and the associative. They are actually four laws since each can be applied to both addition and multiplication.

The commutative law or property with respect to addition states that the order of addends (numbers to be added) is immaterial; the sum will be unaffected. It doesn't matter whether we add 7 and 3 by writing $7 + 3$ or $3 + 7$. Stated symbolically, for any number x and y, $x + y = y + x$.

The associative law or property with respect to addition states that the order in which addends are grouped or associated is immaterial; again the sum will be unaffected. This pertains to a problem with at least three addends. Addition is a binary operation meaning that only two numbers can be added at one time, although their sum may be added to another and so on. In the problem $4 + 5 + 8$, it doesn't matter whether we add 4 and 5 and then add this sum to 8, whether we add 4 and 8 and this sum to 5, or whether we add 5 and 8 and this sum to 4. The answer will always be 17. Stated symbolically: For any numbers x, y, and z, $(x + y) + z = x + (y + z) = (x + z) + y$.

The commutative law for multiplication is similar to the law for addition. It states that the order of factors (numbers to be multiplied) does not affect the product. In the problem 8×7, it doesn't matter whether you multiply 8×7 or 7×8, the result is 56. Stated symbolically: For any numbers x and y, $x \cdot y = y \cdot x$ (raised dot indicates multiplication).

The associative law for multiplication is also similar to the law for addition. It states that in any multiplication problem the product is unaffected by the combinations by which the factors are grouped. Again as with addition, multiplication is binary and the grouping will be in pairs. In the problem $4 \times 9 \times 3$, the same product will be found by $(4 \times 9) \times 3$, or by $4 \times (9 \times 3)$. Stated symbolically: For any numbers x, y, and z, $(x \cdot y) \cdot z = x \cdot (y \cdot z) = (x \cdot z) \cdot y$.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

4. The addition $27 + 31$ can be written $31 + 27$ because of the _____ law.

   a. commutative
   b. associative
   c. reflexive
   d. transitive

5. Which item below illustrates the associative property for addition?

   a. $3 + 6 + 9 = 18$
   b. $18 = 9 + 6 + 3$
   c. $9 + 6 + 3 = 9 + 9$
   d. $(3 + 6) + 9 = (9 + 3) + 6$

6. $7 \times 9 = 9 \times 7$ illustrates the _____ for multiplication.

   a. commutative law
   b. symmetric property
   c. distributive property
   d. transitive property

7. Which item below illustrates the associative law for multiplication?

   a. $2 \times 9 \times 3 = 54$
   b. $54 = 2 \times 9 \times 3$
   c. $2 + 9 + 3 = 3 + 9 + 2$
   d. $(3 \times 9) \times 2 = 3 \times (2 \times 9)$

IDENTIFY THE DISTRIBUTIVE PROPERTY FOR MULTIPLICATION

Distributive property. This law or property states that multiplication can be distributed over addition. It can probably be best explained by use of an example.

Multiply 3 times the sum of 4 and 6. One method of doing the problem is to add the 4 and the 6 then multiply this by 3 to get the answer 30.

$3 \times (4 + 6)$

3×10

30

By the distributive method we distribute the 3 to each of the addends:

$3 \times (4 + 6)$

$(3 \times 4) + (3 \times 6)$

$12 + 18$

30

Although it may not seem like the best method for this particular problem, it is one of the most used properties in algebra. Remember that this property is for multiplication over addition. The converse is not true; addition is not distributive over multiplication. Compare the example shown below with the previous example. Try several to prove it yourself.

$3 + (4 \times 6)$		$3 + (4 \times 6)$
$3 + 24$		$(3 + 4) \times (3 + 6)$
27		7×9
27	does not equal	63

Stated symbolically the distributive law is: For any numbers x, y, and z, $x \cdot (y + z) = x \cdot y + x \cdot z$.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

8. The law that states that multiplication can be distributed over addition is the _____ property.

9. Which statement best describes the example?

$5 \times (2 + 3)$
$(5 \times 2) + (5 \times 3)$
$10 + 15$
25

a. Illustrates the transitive property
b. Illustrates the distributive property
c. Incorrectly illustrates the commutative law
d. Incorrectly illustrates the distributive law

GIVEN SIMPLE EQUATIONS, SOLVE USING THE ZERO AND UNITY ELEMENTS.

Other properties. There are many other laws and properties of numbers and all of them are important for various applications. Because of the narrow scope of this course, there are just two additional ones that should be covered here.

Zero element. As pointed out earlier, the invention of zero by the Hindus is what made the Hindu-Arabic number system. Its value as a place holder and a signifier of a nothing value is what set the system apart from others. The zero has several unique properties that come into play in operations with numbers. The first one is in addition. Zero is referred to as the additive identity or the identity element for addition. This simply means that when zero is added to a number it preserves the identity of the number: $5 + 0 = 5$, $0 + 8 = 8$, $10 + 0 = 10$, etc. A very fundamental, but for some reason not always obvious property.

Multiplication with the zero is quite another story. Where adding zero to a number preserves its identity, multiplication of anumber by zero destroys its identity, for any number times zero is zero. Stated symbolically: For any number x, $x \cdot 0 = 0 \cdot x = 0$.

There are two aspects of division with zero. Dividing zero by a number always results in zero. Dividing a number by zero is impossible. These statements can be proved by several examples. In dividing zero by a number, $\frac{0}{8} = ?$, whatever number replaces the question mark must be a number that when multiplied by 8 will give zero. Remember that in order to check a division problem, multiply the quotient (answer) times the divisor. This will give the dividend. In the above problem, the only possible replacement for the question mark is zero since it is the only number that multiplied by 8 will give zero. You should see that this will be the same for any number divided into zero. Stated symbolically: For any number x, $\frac{0}{x} = 0$. In dividing zero into a number, $\frac{6}{0} = ?$, you should see that there is no replacement for the question mark that when multiplied by zero will give 6. This would be the same for any number. This operation is said to be impossible or undefined. Stated symbolically: For any number x, $\frac{x}{0}$ = impossible.

Unity element. The number one has many significant properties. One aspect that we want to examine is its identity quality. Where zero is the identity element in addition, one is the identity element in multiplication. That is, any number multiplied by one will be equal to that number, or one will preserve the number's identity. This property is sometimes referred to by the important-sounding title, multiplicative identity. Stated symbolically: For any number x, $1 \cdot x = x \cdot 1 = x$.

Division with the number one also has some important aspects. Dividing a number by one preserves its identity in the same manner as multiplying by one. That is, any number divided by one gives us that number. Stated symbolically: For any number x, $\frac{x}{1} = x$.

Dividing one by a number is quite a different story. One divided by a number gives the reciprocal of the number. This is a very important term that you will find throughout your mathematical education. We will have occasion to use it several times in this course. An example: The reciprocal of 15 is $\frac{1}{15}$, the reciprocal of $\frac{3}{4}$ is $\frac{1}{\frac{3}{4}}$ or $\frac{4}{3}$. An important fact to note is that the product of a number and its reciprocal is one: $4 \times \frac{1}{4} = 1$, $15 \times \frac{1}{15} = 1$, $\frac{3}{4} \times \frac{4}{3} = 1$, etc.

EXERCISE: Simplify each of the following problems and check your answer in the back of the study unit. study unit.

10. $\frac{16}{0} = ?$

11. $\frac{7}{1} = ?$

Work Unit 1-6. SYMBOLS OF GROUPING

GIVEN SIMPLE EQUATIONS USING SYMBOLS OF GROUPING, SIMPLIFY THE MATHEMATICAL STATEMENTS.

The symbols of grouping are the punctuation marks of mathematics. They change the meaning of a group of numbers the same way that commas can change the meaning of a group of words. For example, punctuate this sentence. Pvt BUTTPLATE said the Gunny is a meathead. If you are a Gunnery Sergeant you probably did it something like this: "Pvt BUTTPLATE," said the Gunny, "is a meathead." On the other hand you may have completely changed the meaning by doing something like this: Pvt BUTTPLATE said, "The Gunney is a meathead." You should see from this that the placing of punctuation marks is quite important. The same is true for mathematical "sentences". Placing of punctuation marks gives clear indications of the intent of a problem. In math, the punctuation marks are called symbols of inclusion or grouping. There are four <u>different</u> devices used:

(1) parenthesis $(6 - 2)$

(2) bracket $[6 - 2]$

(3) brace $\{6 - 2\}$

(4) bar or viculum $\overline{6 - 2}$

Let us look at a problem and see how the use of one of these devices can clear it up. What is the answer to this problem: $3 \times 7 + 4 = ?$ Is it 25 or 33? There is a proper order of operations for this type of problem, and we shall look into this in the next work unit, but see how the problem is clarified when we insert a grouping symbol such as a parenthesis: $3 \times (7 + 4) = 33$. Moving the parenthesis to another position will produce a different result: $(3 \times 7) + 4 = 25$. In the first example, $3 \times (7 + 4)$, we have a multiplication sign included. When we use the various grouping symbols there is usually no need to include multiplication symbols. Whenever a number directly precedes or follows a grouping symbol, multiplication is indicated. For example, 3(6), (3) (6), and (3)6 all indicate to multiply 3 times 6.

The grouping symbols mentioned above are all used in the same manner. Our original problem could be written in four ways:

$3(7 + 4)$ $\quad$ $3\,[7 + 4]$ $\quad$ $3\,\{7 + 4\}$ $\quad$ $3\,\overline{7 + 4}$

EXERCISE: Simplify each of the following mathematical statements and check your answer in the back of the study unit.

1. $(8 + 2) + 7$
2. $5 + [3 \times 2]$
3. $8 + (3 + 5)$
4. $4(3 + 7)$
5. $2\,\{6 + 5\}$
6. $(8 + 3) + 5$
7. $2(5 + 3)$
8. $[4 \times 3] + [4 \times 7]$
9. $17\,\overline{15 \times 2}$
10. $\{17 + 3\}\,18$

Work Unit 1-7. ORDER OF OPERATIONS

USE THE RULES FOR ORDER OF OPERATIONS TO SOLVE GIVEN EQUATIONS

In solving a mathematical expression it is possible to have more than one answer, but there is only one <u>correct</u> answer. It depends on the order in which you perform the indicated operations of the problem ($3 \times 7 + 4 = 25$ or 33?). In the preceding work unit we saw how the use of grouping symbols gives a clear indication of the intent of the problem, making it easier to work the problem. Whether or not there are any grouping symbols in a problem, there are rules which dictate the proper sequence of operation. These rules must be followed if you are to arrive at the correct answer. They are:

(1) When grouping symbols are used, simplify these parts of the expression <u>first</u>, following the order of operation in rules (2) and (3) listed below.

(2) Perform all multiplication or division operations indicated in the expression in the order you come to them, working from left to right.

(3) Perform all addition or subtraction operations indicated in the expression in the order you come to them, working from left to right.

Remember, when you have a problem or equation which contains more than one type of grouping symbol, "work from the inside out". This simply means complete all of the indicated operations within the innermost set of symbols first, then the next innermost set of symbols second, and so on. When all of the indicated operations within a grouping symbol have been completed, the symbol may be removed from the problem. This will keep the problem from becoming cluttered with unnecessary symbols which could cause you to make an error. Also, neatness and an orderly step-by-step procedure for simplifying a problem will prevent many errors. Remember, you must do all multiplication and division before any addition and subtraction, and the rules must be applied to each step needed to simplify the problem or equation.

Now that we have rules to govern the order of operations, the answer to the expression presented previously is quite easy: $3 \times 7 + 4 = ?$

$21 + 4 = 25$

Let us look at several other examples:

$6 \times 8 - 7 \times 2$	is $(6 \times 8) - (7 \times 2) = 48 - 14 = 34$
$30 \div 10 \times 3$	is $(30 \div 10)3 = 3 \times 3 = 9$
$5 + 3 \times 2$	is $5 + (3 \times 2) = 5 + 6 = 11$
$18 - 6(3) + 5$	is $18 - 18 + 5 = 0 + 5 = 5$
$5 - 2 + 4 \times 3$	is $5 - 2 + (4 \times 3) = 5 - 2 + 12 = 3 + 12 = 15$
$21 + 5(7 + 3) - 20$	is $21 + 5(10) - 20 = 21 + 50 - 20 = 51$

$3\ [(54 \div 18) + (81 + 27) \div 9]$ is $3\ [(3) + (108) \div 9] = 3\ [(3) + (12)] = 3\ [(15)] = 45$

or it could be written

$3\ [3 + (108) \div 9] = 3\ [3 + 12] = 3\ [15] = 45$

EXERCISE: Simplify the following expressions and check your results in the back of the study unit.

1. $(7 + 3 + 2) \div 3 + 1$
2. $(7 + 3 + 2) \div (3 + 1)$
3. $5(7 + 9) \div 4 + 3$
4. $7 + 3(5 - 1) \div 6$
5. $8 \div 2 + 6 \div 3$
6. $30 - 3(7 - 2)$
7. $27 + 10 - (11 + 3)$
8. $(2 \times 3 - 12 \div 3) \div 2$
9. $6(6 + 8) \div 8 - 32 \div 16$
10. $(15 - 3 + 8 \div 2) \div 8 \times 5$

If you miss more than three of the questions, read work units 1-6 and 1-7 again. Look carefully at the examples presented and go over the problems you missed. Remember the rules: Remove the grouping symbols, multiply or divide, then add or subtract.

SUMMARY REVIEW

In the preceding work units you have been introduced or reintroduced in some cases, to the history of the real numbers. You have also identified the natural numbers, integers, and the rational numbers within the real number system. You have determined that there are laws and properties that pertain to the number system. Try to remember the laws and their names and how they are used with the basic operation of numbers. You can now recognize the symbols of grouping and solve expressions that use them following the rules for order of operations.

Answers to Study Unit #1 Exercises

Work Unit 1-1.

1. c. Use of zero
2. b. 0
3. place
4. d. PFC HERO took 10 grenades. (The number 10 represents how many.)

Work Unit 1-2.

1. one
2. c. 3, 15, 27 (Response c is the only item that contained natural numbers. All others contained either a decimal, a fraction, or zero number.)
3. natural
4. b. 4 + 9 = (The answer to 4 + 9 is 13 which is a natural number.)

Work Unit 1-3.

1. c. 8 - 12 (The answer to 8 - 12 is -4 which is an integer.)
2. negative

Work Unit 1-4.

1. fractions
2. rational

Work Unit 1-5.

1. b. Symmetric property (This states that an equality is reversible.)
2. a. Transitive property (This makes it possible to identify two numbers as being equal if each of them is equal to a third quantity.)
3. c. a = a (Any number reflects itself or is equal to itself.)
4. a. commutative (The order of addends is immaterial; the sum will be unaffected.)
5. d. (3 + 6) + 9 = (9 + 3) + 6 (The way the addends are grouped or associated is immaterial; the sum will be unaffected. This pertains to a problem with at least three addends.)
6. a. commutative law (The order of factors doesn't affect the product.)
7. d. (3 x 9) x 2 = 3 x (2 x 9) (In any multiplication problem the product is unaffected by the combinations by which the factors are grouped.)
8. a. distributive
9. b. Illustrates the distributive property
10. a. Impossible (Dividing a number by 0 is impossible.)
11. c. 7 (Any number divided by one gives you that number.)

Work Unit 1-6.

1. $(8 + 2) + 7$
   $4 + 7$
   11

2. $5 + [3 \times 2]$
   $5 + 6$
   11

3. $8 + (3 + 5)$
   $8 + 8$
   16

4. $4(3 + 7)$
   $4(10)$
   40

5. $2\ \{6 + 5\}$
   $2\ \{11\}$
   22

6. $(8 + 3) + 5$
   $11 + 5$
   16

7. $2(5 + 3)$
$2(8)$
16

8. $[4 \times 3] + [4 \times 7]$
$12 + 28$
40

9. $17\ \overline{15 \times 2}$
$17\ 30$
510

10. $\left\{17 + 3\right\}\ 18$
$\left\{20\right\}\ 18$
360

Work Unit 1-7.

1. $(7 + 3 + 2) \div 3 + 1$ — Rule #1
$12 \div 3 + 1$ — Rule #2
$4 + 1$ — Rule #3
5

2. $(7 + 3 + 2) \div (3 + 1)$ — Rule #1
$12 \div 4$ — Rule #2
3

3. $5(7 + 9) \div 4 + 3$ — Rule #1
$5(16) \div 4 + 3$ — Rule #2
$80 \div 4 + 3$ — Rule #2
$20 + 3$ — Rule #3
23

4. $7 + 3(5 - 1) \div 6$ — Rule #1
$7 + 3(4) \div 6$ — Rule #2
$7 + 12 \div 6$ — Rule #2
$7 + 2$ — Rule #3
9

5. $8 \div 2 + 6 \div 3$ — Rule #2
$4 + 2$ — Rule #3
6

6. $30 - 3(7 - 2)$ — Rule #1
$30 - 3(5)$ — Rule #2
$30 - 15$ — Rule #3
15

7. $27 \div 10 - (11 + 3)$ — Rule #1
$27 \div 10 - 14$ — Rule #3
$37 - 14$ — Rule #3
23

8. $(2 \times 3 - 12 \div 3) \div 2$ — Rule #1
$(6 - 4) \div 2$ — Rule #1
$2 \div 2$ — Rule #2
1

9. $6(6 + 8) \div 8 - 32 \div 16$ — Rule #1
$6(14) \div 8 - 32 \div 16$ — Rule #2
$84 \div 8 - 32 \div 16$ — Rule #2
$84 \div 8 - 2$ — Rule #3
80

10. $(15 - 3 + 8 \div 2) \div 8 \times 5$ — Rule #1
$(15 - 3 + 4) \div 8 \times 5$ — Rule #1
$(12 + 4) \div 8 \times 5$ — Rule #1
$16 \div 8 \times 5$ — Rule #2
2×5 — Rule #2
10

STUDY UNIT 2

FRACTIONS AND PERCENTS

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO REDUCE FRACTIONS AND PERFORM THE BASIC OPERATIONS WITH COMMON AND DECIMAL FRACTIONS. YOU WILL ALSO APPLY THE PRINCIPLES OF FRACTIONS TO PERCENTAGE.

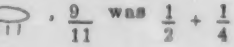
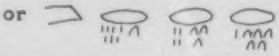
Once upon a time our friend Charlie Caveman and his number one son Calvin were snooping and pooping through the boonies hunting for a tasty brontosaurus for Momma to cook for the evening meal. Calvin spotted one and took up a position with good observation and field of fire while Charlie enveloped to the right. Upon arriving at the assault position, Charlie heard a clattering noise and saw Calvin had opened up the base of fire with his slingshot a little too early. Charlie, weary from his envelopment through the heavy brush, charged out and threw his M1 spear at the brontosaurus. His aim was not steady (poor sight picture) and he missed the beast. His spear hit a rock and broke in two. Charlie sat down to catch his breath while the bumbling Calvin went to retrieve the spear. He returned and said, "Dad, your spear broke. Here is the big half. The little half is splintered." Charlie's ears perked up at this unfamiliar terminology. He though a moment and said, "Calvin you chased away our evening meal, but you have just invented fractions."

Of course this is fantasy, but it is used to show that even primitive man, although he primarily used his limited number of model groups, had an awareness that whole things had parts. Undoubtedly, things such as parts of a journey, portions of food, or part of his herd were recognized for what they are. Undoubtedly, also, it took a more sophisticated culture than Charlie's to actually invent fractions.

With the advent of measurement, particularly for real estate and grain, there came more of a need to know about parts of things. The early civilized cultures had advanced to a fair degree with the natural numbers, but their work with fractions was quite crude and basic. The primary block to establishing an efficient system of fractions was the twofold nature of a fraction. First, it is part of some other entity. Secondly, although it is part of this larger thing, it still an entity itself. For example, if you own a piece of ground that is $\frac{2}{3}$ of an acre in size, you have something that in terms of measurement is part of something larger. But you know that this lot is still a whole thing in its own right.

Some of the early systems of numeration, such as the Egyptain and Roman that were discussed in study unit 1, had some basic symbolism to indicate the idea of fractions. Each of these systems recognized some fundamental aspect of fractions, but each lacked elements to make it a complete functioning system. The Egyptians, for example, had only fractions that had one for a numerator. (Remember that the numerator is the part above the line and the denominator is the part below the line.) The only exception to this was the often-used fraction $\frac{2}{3}$.

The basic Egyptian symbol for a fraction was . This was written over the appropriate number. For example the fraction $\frac{1}{4}$ was written , $\frac{1}{10}$ was , $\frac{1}{20}$ was , $\frac{1}{234}$ was . (Note that all these fractions have a numerator of one). There were two exceptions to this symbolism. The often-used fraction $\frac{1}{2}$ was written and the above-mentioned $\frac{2}{3}$ was written as . Any fraction with a numerator other than one had to be shown as a set of fractions all having numerators of one. For example,

$\frac{3}{4}$ was $\frac{1}{2} + \frac{1}{4}$ or , $\frac{9}{11}$ was $\frac{1}{2} + \frac{1}{4} + \frac{1}{22} + \frac{1}{44}$ or  , and $\frac{31}{51}$ and $\frac{1}{2} + \frac{1}{17} + \frac{1}{34} + \frac{1}{51}$ or 

Contrary to the Egyptian system, the Romans allowed the numerator to vary, but kept a constant denominator of 12. Again the often-used $\frac{1}{2}$ had a special symbol, S $\left(\frac{6}{12}\right)$. The basic symbol $\frac{1}{2}$ was —, $\frac{3}{12}$ was = –, $\frac{8}{12}$ was S =, etc.

Again the Hindus came to the rescue to give some logic to the problem and originate the system that we have today. They took the good qualities from both the Egyptian and Roman systems, eliminated some of the poor qualities, and added a new vertical type of notation. Their system allowed for simultaneous variation in both the number of parts (numerator) and the size of the parts (denominator). (The Egyptians had the former, the Romans had the latter.) The original notation consisted of simply writing one number over the other: $\frac{3}{4}$. The Arabs added the bar between the numerals, $\frac{3}{4}$, as we have it today.

Section I. BASIC PRINCIPLES OF FRACTIONS

Work Unit 2-1. REDUCE FRACTIONS TO LOWEST TERMS

GIVEN A FRACTION, USE THE PROPER OPERATIONS TO REDUCE IT TO ITS LOWEST TERMS.

The golden rule. The basic principle involving the manipulation of fractions is referred to by many textbooks as the "golden rule of fractions." This is: If the numerator and denominator of a fraction are both multiplied or divided by the same number (other than zero), the value of the fraction is not changed. This principle is fundamental to all of the operations with fractions and there are two important points to be examined concerning it. First, the golden rule implies that you must remember to multiply or divide both terms of the fraction. If just the numerator is multiplied, for example, the resulting fraction will not be equal to the initial one. Second, this principle concerns multiplication and division only. If the same number is added to or subtracted from both the numerator and denominator, the value of the resulting fraction will not be the same as the initial one. The importance of this principle lies in the fact that it is often so necessary to change the form of a fraction without changing its value. This change in form is known as a reduction. Strangely enough, the term reduction does not necessarily mean to make smaller, because in mathematics we can reduce to lowest terms or reduce to higher terms. The latter is actually a contradiction, but this is the mathematical terminology. Remember - A reduction is a change in form without a change in value.

Reducing to lowest terms. This is a familiar process that is accomplished by dividing each term of a fraction by a common factor; a process usually referred to as "cancellation." A caution on the use of this word. It has come along with the development of mathematics, but it does not accurately describe what is taking place during its use. Cancel implies to eliminate or get rid of, to leave nothing. Our mathematical cancel will always leave at least a value of 1.

Let us look at some examples of reducing to lowest terms.

$$\frac{4}{6} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{2}{3}$$

Note that 4 and 6 have the common factor 2.

This middle step is usually mental, but it is inserted here to illustrate what is actually being done.

Reduce $\frac{15}{30}$ to lowest terms.

Suppose 5 is used as the common factor. In this case dividing each term by 5 gives

$$\frac{15}{30} = \frac{\cancel{15}^{3}}{\cancel{30}_{6}} = \frac{3}{6}$$

Here though $\frac{3}{6}$ is not expressed in lowest terms; there is still a common factor 3.

$$\frac{3}{6} = \frac{\cancel{3}^{1}}{\cancel{6}_{2}} = \frac{1}{2}$$

The highest common factor, 15, would have given the quickest reduction, but sometimes the highest common factor is not readily evident. Always express an answer in lowest terms. When all common factors, other than 1, have been removed from the numerator and denominator, the fraction is in lowest terms.

EXERCISE: Reduce the fractions to lowest terms and check your answers in the back of the study unit.

1. $\frac{56}{48}$
2. $\frac{28}{56}$
3. $\frac{78}{102}$
4. $\frac{99}{144}$
5. $\frac{186}{378}$
6. $\frac{680}{2520}$

Work Unit 2-2. REDUCE FRACTIONS TO HIGHER TERMS

GIVEN A FRACTION, USE THE PROPER OPERATIONS TO REDUCE IT TO ITS HIGHER TERMS.

Reducing to higher terms. This is the opposite of the operation describe in work unit 2-1. Instead of dividing by a common factor, multiply the numerator and denominator by some number greater than one. Reversing the procedure with our examples above:

$$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{15 \times 1}{15 \times 2} = \frac{15}{30}$$

The middle step is usually mental.

It should be evident that to change a fraction to some desired higher term, you must divide the denominator of the smaller fraction into the denominator desired. This quotient is the common factor. For example, change $\frac{2}{5}$ to an equivalent fraction with a denominaotr of 30.

$$\frac{2}{5} = \frac{?}{30}$$

What number must be multiplied times 5 in order to get 30? Since you have memorized the primary multiplication combinations, you know instantly that it is 6. Therefore multiply the numerator by 6 and get:

$$\frac{2}{5} = \frac{12}{30}$$

When the desired denominator reaches a size larger than a product of the primary combinations, then divide as mentioned above.

As you know, the resulting fraction is the equivalent of the initial fraction although the form has changed. The ability to reduce to higher terms is essential for adding and subtracting fractions.

EXERCISE: Reduce the following fractions to higher terms by supplying the missing numerators. Check your answers in the back of the study unit.

Example: $\frac{3}{5} = \frac{?}{100}$

$\frac{3}{5} = \frac{20 \times 3}{20 \times 5} = \frac{60}{100}$

1. $\frac{4}{9} = \frac{?}{27}$
2. $\frac{3}{4} = \frac{?}{100}$
3. $\frac{7}{8} = \frac{?}{240}$
4. $\frac{5}{7} = \frac{?}{56}$
5. $\frac{5}{12} = \frac{?}{132}$
6. $\frac{9}{37} = \frac{?}{111}$

Section II. OPERATIONS WITH FRACTIONS

Work Unit 2-3. ADDITION OF FRACTIONS

GIVEN A SERIES OF FRACTIONS, USE THE OPERATIONS FOR ADDITION OF FRACTIONS TO FIND THE SUM.

Addition. Recall from study unit 1 that the operation of addition of natural numbers is based on several principles, among them the commutative and associative laws and likeness. The addition operation with the rational numbers is also based on these same principles. Of particular concern is the principle of likeness which is controlled by the position of the numerals. In the natural numbers, recall that we add the numbers in the same columns. The column itself provides the likeness: first column is units, second column is tens, third is hundreds, etc. The determining factor of likeness in fractions is the size of the parts to be combined. Of course you know that the size of the part is the denominator. For example, the fractions $\frac{4}{5}$ and $\frac{4}{9}$, although both 4 of something, are not alike. However the fractions $\frac{3}{5}$ and $\frac{1}{5}$ are alike because although the number of parts is different, the size of the parts is the same. When this is so, there is no difficulty in adding the fractions. The number of parts (numerators)are added to find the total number being considered. Symbolically we could characterize addition of fractions as: For any numbers a, b, and c (except $b = 0$),

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

A practical example would be to add:

$$\frac{3}{12} + \frac{2}{12} + \frac{1}{12} = \frac{3+2+1}{12} = \frac{1}{2}$$

The middle step is not necessary. It is included to show the mental step that is taken. Note the answer is expressed in lowest terms.

Now let us examine the operation of adding fractions with unlike denominators. Take for example the addition of $\frac{1}{3}$ and $\frac{5}{6}$. Clearly, they are not alike. Something must be done to one or both of these fractions to make them alike. Remember that it was mentioned that the process of reduction would be necessary in adding and subtracting fractions. We want to change the form of either $\frac{1}{3}$ or $\frac{5}{6}$. It is easier to change the $\frac{1}{3}$ to sixths without changing the value of the fraction. This is a problem similar to the ones in the last exercise.

$$\frac{1}{3} = \frac{?}{6}$$

Clearly the answer is $\frac{2}{6}$. Now it is quite simple to add $\frac{2}{6} + \frac{5}{6} = \frac{7}{6} = 1\frac{1}{6}$

Again note that the answer has been reduced to lowest terms. (As a review, what kind of fraction is $\frac{7}{6}$? How about $1\frac{1}{6}$? If you said improper and mixed number, you are correct.)

Add $\frac{1}{2}$ and $\frac{3}{8}$

First $\frac{1}{2}$ should be changed to an equivalent fraction with a denominator of 8. This is $\frac{4}{8}$ which can be added to $\frac{3}{8}$

$$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

How about when one denominator cannot be reduced to the size of the other one? What is done then? For example,

$$\frac{1}{5} + \frac{3}{4} = ?$$

The fraction $\frac{1}{5}$ cannot be expressed as fourths and $\frac{3}{4}$ cannot be expressed as fifths. As you know a denominator must be found to which each of these denominators can be reduced (a common denominator). The quickest way to find a common denominator is to find the product of the denominators of the fractions to be added. For example, the common denominator for $\frac{1}{5}$ and $\frac{3}{4}$ is 20. This simply means that each of these fractions can undergo a reduction to 20.

$$\frac{1}{5} = \frac{4}{20} \qquad \frac{3}{4} = \frac{15}{20}$$

$$\therefore \quad \frac{1}{5} + \frac{3}{4} = \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$$ (The symbol $\therefore$ is read, therefore)

What is a common denominator of $\frac{1}{6}$ and $\frac{3}{4}$? If you said 24, you are correct and the fractions would be reduced in this manner:

$$\frac{1}{6} + \frac{3}{4} = \frac{4}{24} + \frac{18}{24} = \frac{22}{24} = \frac{11}{12}$$

Notice that although 24 is a common denominator, there is a better one that could be used. Each of the denominators 6 and 4 can be reduced to 12. This you will remember is called the least or lowest common denominator (LCD). See how the problem is worked using this:

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

The last step, reducing to lowest terms, was eliminated by selecting the LCD. Always try to find the LCD, but remember that the product of all of the denominators is a common denominator and can be used. The main disadvantage of this is that you will sometimes have to work with larger numbers and then reduce to lowest terms at the end. For those who like definitions and rules: The sum of two fractions with different denominators is obtained by replacing these fractions with equivalent fractions having a common denominator, and then adding the numerators.

Symbolically, for any fractions

$$\frac{a}{b} \text{ and } \frac{c}{d}, \quad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Note that throughout this work unit addition has been performed in a horizontal form. It may just as well, and sometimes should, be done in a vertical form.

Examples:

$$3\frac{2}{7} = 3\frac{4}{14}$$
$$+\frac{3}{14} = \frac{3}{14}$$
$$3\frac{7}{14} = 3\frac{1}{2}$$

$$9\frac{3}{4} = \frac{3}{4}$$
$$+1\frac{1}{2} = 1\frac{2}{4}$$
$$10\frac{5}{4} = 11\frac{1}{4}$$

EXERCISE: Add the following fractions and check your results in the back of the study unit.

1. $\frac{3}{8} + \frac{1}{3}$

2. $\frac{1}{9}$
$+\frac{1}{6}$

3. $\frac{2}{7}$
$+\frac{1}{12}$

4. $\frac{3}{8} + \frac{7}{20}$

5. $\frac{3}{8}$
$\frac{1}{12}$
$\frac{2}{15}$
$+\frac{3}{20}$

6. $7\frac{7}{8}$
$2\frac{1}{3}$
$+\ 3\frac{5}{6}$

7. $\frac{1}{6} + \frac{1}{12} + \frac{1}{9} + \frac{1}{8}$

8. $3\frac{3}{7} + 4\frac{11}{63} + 10\frac{4}{9}$

Work Unit 2-4. SUBTRACTION OF FRACTIONS

GIVEN TWO FRACTIONS, USE THE OPERATIONS FOR SUBTRACTION OF FRACTIONS TO FIND THE DIFFERENCE.

Subtraction. As stated in study unit 1, subtraction is the opposite of addition. Since the rules of addition of natural numbers apply to rational numbers, it follows that the rules for subtraction of natural numbers apply to subtraction of rational numbers. In particular, pay attention to the principle of likeness which states that only like fractions may be subtracted. This as you know means that only fractions having like denominators may be subtracted. This is done by subtracting the numerators and expressing the result over the common denominator.

$$\frac{11}{15} - \frac{7}{15} = \frac{11 - 7}{15} = \frac{4}{15}$$

For fractions with unlike denominators, the process again is similar to addition. Find the LCD and express each fraction as an equivalent fraction with the LCD. Subtract the numerators and reduce the resulting fraction to the lowest terms. For example:

$$\frac{7}{12} - \frac{1}{6} = \frac{7}{12} - \frac{2}{12} = \frac{5}{12}$$

$$\frac{7}{12} - \frac{3}{8} = \frac{14}{24} - \frac{9}{24} = \frac{5}{24}$$

There is one other facet of subtraction that should be covered here; that is subtracting of mixed numbers. At times the process of "borrowing" presents a problem to students. Let us look at several examples. First fractions with like denominators, and no borrowing.

$$\begin{array}{r} 15\frac{2}{3} \\ -\,6\frac{1}{3} \\ \hline 9\frac{1}{3} \end{array}$$

Here the numerators were subtracted and placed over the common denominator and then the intergers were subtracted. Now how about

$$\begin{array}{r} 15\frac{1}{6} \\ -\,12\frac{5}{6} \\ \hline \end{array}$$

Here the fractional part of the subtrahend (the number being subtracted) is larger than the fractional part of the minuend. This is solved by borrowing 1 from 15, converting it to its fractional form ($1 = \frac{6}{6}$) and adding it to $\frac{1}{6}$:

mental

$$\begin{array}{rcl} 15\frac{1}{6} & = & 14 + \frac{6}{6} + \frac{1}{6} = 14\frac{7}{6} \\ -\,12\frac{5}{6} & & \quad 12\frac{5}{6} \\ \hline & & \hline \\ & & 2\frac{2}{6} = 2\frac{1}{3} \end{array}$$

How about another one?

$$\begin{array}{rcl} 4\frac{3}{8} & = & 4\frac{3}{8} = 3\frac{11}{8} \\ -\,2\frac{3}{4} & = & 2\frac{6}{8} = 2\frac{6}{8} \\ \hline & & \hline \\ & & 1\frac{5}{8} \end{array}$$

Note that $\frac{3}{4}$ was converted to an equivalent fraction with the common denominator 8. This still left the fraction of the subtrahend smaller than the minuend. One was borrowed from 4 in the fractional form $\frac{8}{8}$ and added to $\frac{3}{8}$. This gave a workable fraction which could then be subtracted in the normal manner.

A definition of subtraction of fractions has been presented. Symbolically it can be represented as: For any two fractions

$$\frac{a}{b} \text{ and } \frac{c}{d}, \text{ if } ad > bc, \text{ then}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

(Note the symbol $>$ is read: is greater than)

EXERCISE: Subtract the following and check your answers in the back of the study unit.

1. $\begin{array}{r} \frac{7}{9} \\ -\ \frac{1}{6} \\ \hline \end{array}$

2. $\begin{array}{r} 3\frac{1}{3} \\ -\ \frac{5}{6} \\ \hline \end{array}$

3. $\begin{array}{r} 2\frac{3}{8} \\ -\ \frac{5}{8} \\ \hline \end{array}$

4. $\begin{array}{r} 3\frac{5}{36} \\ -1\frac{4}{9} \\ \hline \end{array}$

5. $\begin{array}{r} 7\frac{5}{12} \\ -\ 2\frac{7}{12} \\ \hline \end{array}$

6. $\begin{array}{r} 3\frac{2}{3} \\ -\ \frac{11}{12} \\ \hline \end{array}$

7. $\begin{array}{r} 5 \\ -\ 2\frac{2}{3} \\ \hline \end{array}$

8. $\begin{array}{r} 2 \\ -\ \frac{49}{64} \\ \hline \end{array}$

Work Unit 2-5. MULTIPLICATION OF FRACTIONS

GIVEN TWO FRACTIONS, USE THE OPERATIONS FOR MULTIPLICATION OF FRACTIONS TO FIND THE PRODUCT.

Multiplication. Here again is an operation that is based on the same principles that applied to the natural numbers: associative, commutative, and distributive laws. Multiplication is still for fractions a rapid or shortened addition process. All one needs to do in order to multiply fractions, therefore, is draw on the principles he has already learned.

There is one aspect of multiplication of fractions that is unique and proves troublesome to the thinking of some students. To multiply usually implies to gain, or to make larger. This is true with integers, but when dealing with multiplication by a fraction, the converse is true; a smaller quantity will result. For example if you take one-half of something, say a dozen eggs, you end up with a smaller quantity, 6 eggs, although you actually performed a multiplication, $\frac{1}{2}$ times 12. The student should keep this fact in mind when multiplying by a fraction and have confidence in the answers he produces.

Multiplying a fraction by a whole number. Multiplication has been defined as a shortened addition of like addends. Therefore, multiplication of fractions is also a shortened addition of like addends. The addends in this case have like numerators and denominators. For example, to add

$$\frac{2}{13} + \frac{2}{13} + \frac{2}{13} + \frac{2}{13},$$

recall that the numerators are added and placed over the like denominator. In this case $\frac{8}{13}$ which you can see is 4 (number of addends) times 2 (like numerator) over the like denominator. Stated as a rule: To multiply a whole number times a fraction, multiply the whole number times the numerator and express this product as the numerator of a new fraction whose denominator is the same as the original fraction. Symbolically: For any numbers a, b, and c,

$$a \cdot \frac{b}{c} = \frac{ab}{c}$$

mental step

Example: $7 \times \frac{3}{5} = \underbrace{\frac{7 \times 3}{5}} = \frac{21}{5} = 4\frac{1}{5}$

One further example by way of a pictorial representation should illustrate the rationale of multiplying an integer times a fraction (fig 2-1).

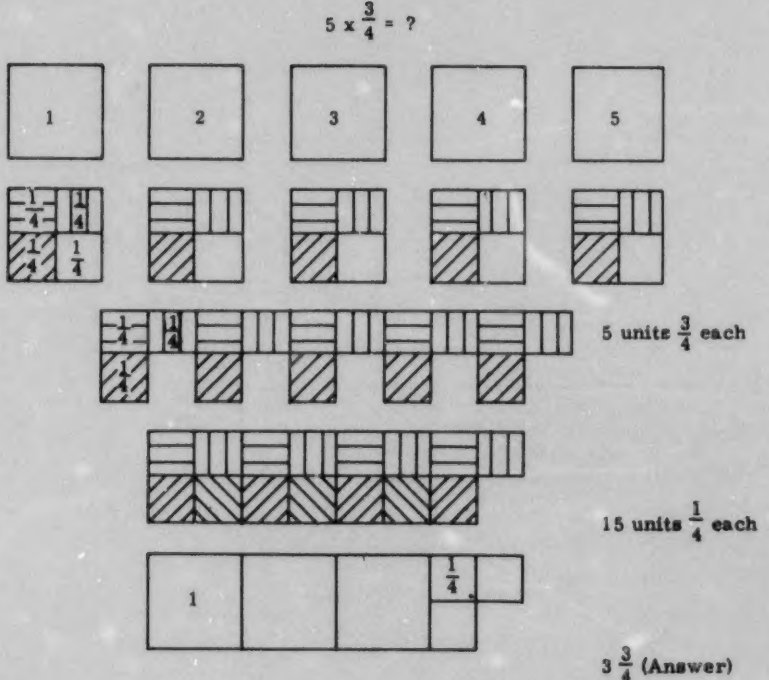


Fig 2-1. An integer times a fraction. Note that this is a putting-together operation.

Multiplying a whole number by a fraction (fig 2-2). Because of the commutative law, you know that this process will produce the same result as multiplying a fraction by a whole number. The mechanics of the operation are the same: Multiply the numerator of the fraction times the integer and express this product as the numerator of a new fraction having the denominator of the original fraction. Symbolically, for any numbers a, b, and c, $\frac{a}{c} \cdot b = \frac{ab}{c}$

Find $\frac{2}{5}$ of 8 acres. $\frac{2}{5} \times 8 = ?$

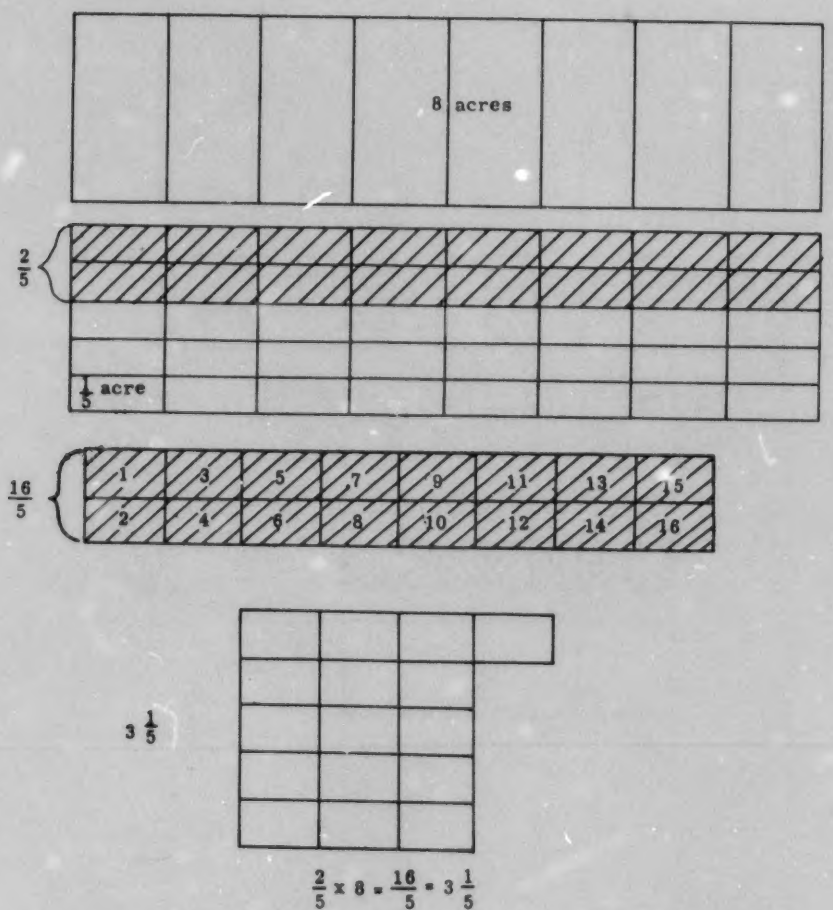


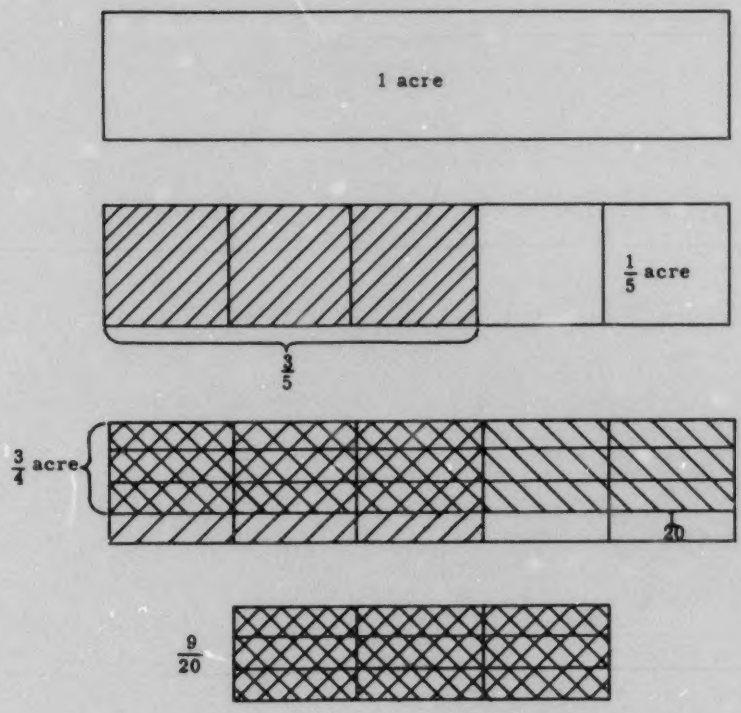
Fig 2-2. A fraction times an integer. Note that this is finding part of some whole quantity.

Multiplying a fraction by a fraction (fig 2-3). In this operation we are interested in finding part of a part. The mechanics of the process are the same as the two already discussed except that the denominators are multiplied also. Symbolically, for any numbers a, b, c, and d (except b and d = 0),

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Find $\frac{3}{4}$ of $\frac{3}{5}$ of an acre.

$$\frac{3}{4} \times \frac{3}{5} = ?$$



$$\frac{3}{4} \times \frac{3}{5} = \frac{9}{20} \text{ of 1 acre}$$

Fig 2-3. A fraction times a fraction. Finding a part of a part.

Cancellation. Earlier in the study unit the term cancellation was used to indicate reducing to lowest terms. Many times the proper use of canceling when multiplying fractions will enable you to work with smaller numbers and therefore find an answer more quickly. Look at the example worked first without canceling and then with canceling.

$$\frac{8}{18} \times \frac{6}{20} = ?$$

$$\frac{8}{18} \times \frac{6}{20} = \frac{48}{360}$$ This must now be reduced to finding the highest common factor.

$$\frac{48 \div 24}{360 \div 24} = \frac{2}{15}$$

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{18}}} \times \frac{\overset{1}{\cancel{6}}}{\underset{5}{\cancel{20}}} = \frac{2}{15}$$ Divide 8 and 20, by 4, and 6 and 18 by 6.

Note that you may cancel the numerator of one fraction with the denominator of another in addition to the normal reducing of the individual fraction. From this you should see that there is a definite advantage to canceling whenever possible before multiplying.

Multiplying with mixed numbers. This involves changing the mixed numbers to improper fractions and then proceeding with the rules for multiplying fractions.

Example: $4\frac{2}{7} \times 5\frac{5}{9} = ?$

$$\frac{\overset{10}{\cancel{30}}}{7} \times \frac{50}{\underset{3}{\cancel{9}}} = \frac{500}{21} = 23\frac{17}{21}$$

EXERCISE: Multiply the following and check your answers in the back of the study unit.

1. $\frac{5}{8} \times 16$
2. $\frac{4}{5} \times 2$
3. $\frac{2}{3} \times \frac{5}{6}$
4. $6\frac{2}{3} \times 15$
5. $\frac{8}{9} \times 10$
6. $8\frac{2}{3} \times 1\frac{1}{2}$
7. $8 \times 2\frac{1}{2}$
8. $7\frac{1}{2} \times \frac{4}{15}$
9. $\frac{15}{16} \times \frac{12}{25}$
10. $5 \times 17\frac{2}{3}$

Work Unit 2-6. DIVISION OF FRACTIONS

GIVEN TWO FRACTIONS, USE THE OPERATIONS FOR DIVISION OF FRACTIONS TO FIND THE QUOTIENT.

Division. There are two computational techniques generally in use for division of fractions: The common denominator method and the simpler and probably more used inverted divisor method. Both will be discussed.

Common denominator. By this method, first a common denominator is found just as in addition and subtraction. Each fraction is reduced to an equivalent fraction having the common denominator. The denominators are then disregarded and the numerators are divided. This can be shown with an example.

$$\frac{3}{4} \div \frac{1}{3} = ?$$

The common denominator is 12. Reduce each fraction to an equivalent fraction with a denominator of 12.

$$\frac{9}{12} \div \frac{4}{12}$$

Divide 9 by 4 and 12 by 12.

$$\frac{\frac{9}{12} \div \frac{4}{12}}{\frac{1}{1}} = 2\frac{1}{4} = 2\frac{1}{4}$$

You should see that the divided denominators will always be equal to 1, therefore this step may be eliminated.

Example: LCD = 16 $\quad \frac{5}{16} \div \frac{5}{8} = ?$

$$\frac{5}{16} \div \frac{5}{8} = \frac{5}{16} \div \frac{10}{16} = \frac{5}{10} = \frac{1}{2}$$

The common denominator method can also be done another way. This is by multiplying the original fractions by the LCD. For example,

$$\left(\frac{3}{4} \div \frac{1}{3}\right) = ?$$

LCD 12

$$\left(\frac{3}{\cancel{12}} \times \frac{3}{\cancel{4}_{1}}\right) \div \left(\frac{4}{\cancel{12}} \times \frac{1}{\cancel{3}_{1}}\right) = 9 \div 4 = 2\frac{1}{4}$$

Another example: $\quad \frac{5}{16} \div \frac{5}{8} = ?$

LCD 16

$$\left(\frac{1}{\cancel{16}} \times \frac{5}{\cancel{16}_{1}}\right) \div \left(\frac{2}{\cancel{16}} \times \frac{5}{\cancel{8}_{1}}\right) = 5 \div 10 = \frac{1}{2}$$

Inverted divisor. This is the method that you probably learned in school. Simply stated, you remember it as invert the divisor and multiply. Let us see why this is done. This method is based on the idea of multiplying by the reciprocal of the divisor. Although this sounds complicated, remember from study unit 1 that the reciprocal of a number is 1 divided by the number, or that the product of the number and its reciprocal is equal to 1. Let us take the examples used above to illustrate.

$$\frac{3}{4} \div \frac{1}{3} =$$

This example is read $\frac{3}{4}$ divided by $\frac{1}{3}$, so the divisor is the right fraction. What is the reciprocal of the divisor?

Right, $\frac{3}{1}$. Now above we said that we would multiply by the reciprocal of the divisor. We must multiply both the divisor and the dividend by this number.

$$\left(\frac{3}{4} \times \frac{3}{1}\right) \div \left(\frac{1}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{1}\right) = \frac{9}{4} \div 1 = \frac{9}{4} = 2\frac{1}{4}$$

Note that the left fraction becomes $\frac{9}{4}$ and the right one becomes 1. This will <u>always</u> be the case with the right fraction divisor. Therefore the right half of the problem can be eliminated. You should see that the left fraction then fits the rule: Invert the divisor and multiply. Another example, first with the extra step and then without it.

$$\frac{5}{16} \div \frac{5}{8} = \left(\frac{\cancel{5}^{1}}{\cancel{16}_{2}} \times \frac{\cancel{8}^{1}}{\cancel{5}_{1}}\right) \div \left(\frac{\cancel{5}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{8}^{1}}{\cancel{5}_{1}}\right) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$\frac{5}{16} \div \frac{5}{8} = \frac{\cancel{5}^{1}}{\cancel{16}_{2}} \times \frac{\cancel{8}^{1}}{\cancel{5}_{1}} = \frac{1}{2}$$

It is hoped that the explanation gives you some insight into why the rule is valid and that it makes it more meaningful to you.

One further example should be discussed: dividing a fraction by an integer.

$$\frac{7}{8} \div 2 =$$

Remember that 2 is actually the fraction $\frac{2}{1}$ so the reciprocal is $\frac{1}{2}$. Don't make the mistake of canceling the 2 with the 8.

$$\frac{7}{8} \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$$

EXERCISE: Divide the following by either method and check your answers in the back of the study unit.

1. $\frac{3}{8} \div \frac{3}{4}$

2. $\frac{3}{7} \div \frac{5}{9}$

3. $\frac{8}{17} \div \frac{3}{5}$

4. $\frac{6}{27} \div \frac{12}{35}$

5. $\frac{14}{27} \div \frac{7}{9}$

6. $\frac{1}{2} \div \frac{7}{6}$

7. $\frac{63}{360} \div \frac{28}{45}$

8. $6\frac{2}{3} \div 2\frac{3}{4}$

9. $114\frac{7}{12} \div 6\frac{7}{8}$

10. $\left(\frac{3}{7} - \frac{1}{14}\right) \div \frac{35}{49}$

Section III. DECIMAL FORM

It may seem odd to some students that in the study unit about fractions we are discussing decimals. On examination, however, you should see that decimals are just another way of writing fractions. Decimal fractions can be converted to common fractions, and vice versa, with very little difficulty.

There is no sure date for the origin of decimals. Our friend Charlie Caveman and his primitive brothers probably gave root to the notion with the realization that they had 10 fingers. There is evidence that the Hindus were aware of decimals and used them to a limited degree. Early Chinese cultures used decimals in studying meteorology. European astronomers used them in the 15th century and a Persian mathematician computed π to 16 decimal places, also in the 15th century. Various methods were used to indicate the location of our decimal point. In fact even today several European countires use a comma and England uses the raised dot that we sometimes use to indicate multiplication.

Decimal numeration (fig 2-4). Our decimal system, as the name implies, is based on the powers of ten. Every number whether to the right or left of the decimal point is $\frac{1}{10}$ of the size of its neighbor to the left. For example, the hundreds column is $\frac{1}{10}$ the size of the thousands column. The units column is $\frac{1}{10}$ the size of the tens column. The first column to the right of decimal point is $\frac{1}{10}$ of the units column or tenths. The next one to the right is $\frac{1}{10}$ of the tenths or one-hundredths. The next one is thousandths, and so on. Note that figure 2-4 is only partial since the scale extends indefinitely in both directions. Also note that the system is symmetrical about the units column. That is, one to the left is tens, one to the right is tenths; two to the left is hundreds, two to the right is hundredths; six to the left is millions, six to the right is millionths. It is important to note that the system is not symmetrical about the decimal point. The primary purpose of the decimal point is to separate the whole and fractional parts of a number and to designate the location of the units digit.

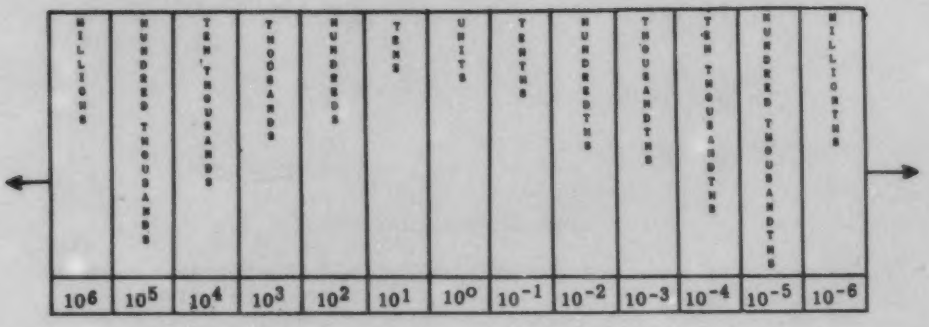


Fig 2-4. Decimal numeration.

A decimal fraction by definition is a fraction whose denominator is a power of ten. There will always be a particular number of tenths, hundredths, thousandths, etc, and never any thirds, eighths, twelfths, etc. Other than having these fixed denominators, the only difference between common fractions and decimal fractions is the form in which they are written (fig 2-5). Also the last digit to the right in a decimal fraction determines the size of the fraction which is the same as the denominator.

$\frac{7}{10}$ = seven tenths = .7

$\frac{39}{100}$ = thirty-nine hundredths = .39

$\frac{421}{1000}$ = four-hundred twenty-one thousandths = .421

$\frac{57}{10000}$ = fifty-seven ten-thousandths = .0057
Note that zeros are needed for their use as place holders.

Fig 2-5. Comparison of form of common fractions and decimal fractions.

Work Unit 2-7. CONVERT DECIMALS TO COMMON FRACTIONS

GIVEN A DECIMAL FRACTION, USE THE OPERATIONS TO CONVERT TO A COMMON FRACTION.

Reductions. Since decimal fractions and common fractions are simply different forms of the same thing, it should come to mind that one can be changed to the other.

Decimal fraction to common fraction. We have already discussed the fact that a decimal fraction has a denominator of some power of ten. For example .8 is the same as $\frac{8}{10}$ which can be reduced to $\frac{4}{5}$. Some will be possible to reduce such as this one, others, such as .33 = $\frac{33}{100}$, will not be able to be reduced further. It is relatively easy to determine by inspection whether a fraction can be reduced further. Since the denominators are all powers of ten, you need only try to divide by 2, 5, or 10 (factors of 10). If the fraction is not divisible by one of these, it is in its lowest terms.

Examples:

$$.15 = \frac{15}{100} = \frac{3}{20}$$

$$.008 = \frac{8}{1000} = \frac{1}{125}$$

$$.025 = \frac{25}{1000} = \frac{1}{40}$$

$$.4 = \frac{4}{10} = \frac{2}{5}$$

$$.88 = \frac{88}{100} = \frac{22}{25}$$

$$.0017 = \frac{17}{10000}$$

EXERCISE: Reduce the following decimal fractions to common fractions in their lowest terms and check your answers in the back of the study unit.

1. .06 2. .278 3. .0075 4. .432 5. .00375 6. .8625

Work Unit 2-8. CONVERT COMMON FRACTIONS TO DECIMALS

GIVEN A COMMON FRACTION, USE THE OPERATIONS TO CONVERT TO A DECIMAL FRACTION.

Common fraction to decimal fraction. This operation is a little more complicated than the one just mentioned. It involves converting the denominator to a power of ten. For example, $\frac{9}{25}$ can easily be converted. What is the most likely power of ten to which 25 could be converted? Did you say 100? Right. Now, what must be done to 25 to convert it to 100? You're right. Multiply it by 4. Don't forget the Golden Rule. If the denominator is multiplied by 4, then the numerator must also be multiplied by 4.

$$\frac{9}{25} = \frac{4 \times 9}{4 \times 25} = \frac{36}{100} = .36$$

Examples:

$$\frac{11}{20} = \frac{5 \times 11}{5 \times 20} = .55$$

$$\frac{17}{125} = \frac{8 \times 17}{8 \times 125} = \frac{136}{1000} = .136$$

$$\frac{3}{5} = \frac{2 \times 3}{2 \times 5} = .6$$

It is hoped that you have noticed that it will not always be possible to multiply the fraction by a whole number to get a power of ten. For example, denominators of 8, 13, 22, and 15 are not factors of integral powers of ten. In cases such as these it is easier to use a different method. This involves dividing the numerator by the denominator. Since this is division of decimals which has not yet been discussed, it will not be covered at this time.

EXERCISE: Reduce the following to decimal fractions by multiplying by a number that will give a denominator that is a power of ten. Check your answers in the bank of the study unit.

1. $\frac{13}{50}$

2. $\frac{8}{25}$

3. $\frac{74}{125}$

4. $\frac{7}{20}$

Section IV. OPERATIONS WITH DECIMALS

Work Unit 2-9. ADDITION OF DECIMALS

GIVEN A SERIES OF DECIMALS, APPLY THE OPERATIONS FOR ADDITION OF DECIMALS TO FIND THE SUM.

Addition. As with integers, the addition process with decimals is based on the principles presented in study unit 1. In particular, again only like things may be added: tenths to tenths, hundredths to hundredths, etc. When addends are lined up in this manner, the decimal points fall in a straight line.

$$\begin{array}{r} 2.18 \\ 34.35 \\ 0.14 \\ 4.90 \\ \hline 41.57 \end{array}$$

The columns are added in the usual order and the decimal point of the sum falls directly below the decimal points of the addends.

EXERCISE: Complete the following and check your answers in the back of the study unit.

1. $\begin{array}{r} 3.68 \\ 4.975 \\ 1.3 \\ \hline 16.42 \end{array}$

2. $\begin{array}{r} 64.2 \\ .04 \\ 18. \\ \hline 17.37 \end{array}$

3. $\begin{array}{r} 8.637 \\ 492. \\ .003 \\ \hline .1 \end{array}$

Work Unit 2-10. SUBTRACTION OF DECIMALS

GIVEN TWO DECIMALS, APPLY THE OPERATIONS FOR SUBTRACTION OF DECIMALS TO FIND THE DIFFERENCE.

Subtraction. This operation also involves no new principles. Place values are again alined which causes the decimal points to aline also. This alinement is also maintained in the answer.

$$\begin{array}{r} 45.76 \\ -\ 31.87 \\ \hline 13.89 \end{array} \qquad \begin{array}{r} 8.64000 \\ -\ .00437 \\ \hline 8.63563 \end{array}$$

EXERCISE: Complete the following items and check your answers in the back of the study unit.

1. 46.37 - 18.48
2. 21.3 - 2.004
3. 307 - 60.42
4. 4.2 - .00005

Work Unit 2-11. MULTIPLICATION OF DECIMALS

GIVEN TWO DECIMALS, APPLY THE OPERATIONS FOR MULTIPLICATION OF DECIMALS TO FIND THE PRODUCT.

Multiplication. Two cases of multiplying decimals will be considered: general multiplication and multiplication by powers of ten.

General. Multiplication of decimals can be explained in two ways. The first is more of a commonsense type of explanation rather than a statement of principles. For example, if you are multiplying 3.2 x 7.6 you must realize that although there are two digits in each of these numbers, part of each is a fraction. The answer will be close to 21 which is the product of the whole numbers 3 and 7.

$$\begin{array}{r} 7.6 \\ 3.2 \\ \hline 152 \\ 228 \\ \hline 24.32 \end{array}$$

Some other examples may be helpful.

$$\begin{array}{r} 4.5627 \\ \times\ 10.37 \\ \hline 319389 \\ 136881 \\ 456270 \\ \hline 47.315199 \end{array}$$

Although the multiplicand has 5 digits and the multiplier has 4, the non-fractional parts of the numbers are 10 and 4. The product then must be close to 40. Therefore the answer is 47.315199.

$$\begin{array}{r} 126.3 \\ \times\ 12.791 \\ \hline 1263 \\ 11367 \\ 8841 \\ 2526 \\ 1263 \\ \hline 1615.5033 \end{array}$$

This is roughly 10 times 125 or 1250 which has 4 digits. Therefore the answer is 1615.5033.

As you can see this method is all right as long as the numbers don't get too large. One thing can be observed from the examples. This is that the answer in each case has as many digits following the decimal point as the sum of the digits following the decimal points in the multiplier and multiplicand. For example, in the last example above, the multiplicand 126.3 has one digit following the decimal point and the multiplier 12.791 has three for a total of four. Note that the answer also has four.

The other explanation of multiplying decimals is by converting the decimal fractions to common fractions.

Examples:

1. .4 x .37

common fractions

$$\frac{4}{10} \times \frac{37}{100}$$

$$\frac{148}{1000}$$

.148

decimal fractions

.37
. 4
.148

2. 4.316 x 3.4

$$\frac{4316}{1000} \times \frac{34}{10}$$

$$\frac{146744}{10000}$$

14 $\frac{6744}{10000}$

14.6744

3.4
4.316
204
34
102
136
14.6744

3. .453 x 78.2

$$\frac{453}{1000} \times \frac{782}{10}$$

$$\frac{354246}{10000}$$

35 $\frac{4246}{10000}$

35.4246

78.2
.453
2346
3910
3128
35.4246

Again the key thing to note is that the number of digits following the decimal point in the answer is the sum of the digits following the decimal points in the multiplier and the multiplicand.

This leads us to the general rule for multiplication of decimals: Ignore the decimal points and multiply as though the multiplier and multiplicand were integers. Locate the decimal point in the product to the left as many places as there are to the right of the decimal points in the multiplier and multiplicand.

Power of ten. This operation is used enough to warrant special comment. By doing enough examples, you can discover for yourself what happens when you multiply by 10, 100, 1000, etc., or by .1, .01, .001, etc. Several examples should be sufficient for you to see what happens to the decimal point.

$10 \times 7 = 70$

$100 \times 94 = 9400$

$100 \times 3.6 = 360$

$1000 \times .5891 = 589.1$

$10 \times .0734 = .734$

$10{,}000 \times 9.8556 = 98556$

You should see that when multiplying by a power of ten, the decimal point is moved to the right as many places as there are zeros in the multiplier.

Another aspect of multiplying by powers of ten is multiplying by .1, .01, .001, etc., which are negative powers of ten. This operation is opposite in effect to the one just described.

$.01 \times 153 = 1.53$

$.001 \times 348.2 = .3482$

$.1 \times 59.996 = 5.9996$

$.0001 \times 13.28 = .001328$

$.01 \times .0177 = .000177$

Note that the effect here is to move the decimal point to the left as many places as there are decimal places (not zeros) in the multiplier.

As mentioned, this operation occurs frequently and you can save time by recognizing the problem and moving the decimal point without multiplying. Practice this and be confident in it and you will save time in computations.

EXERCISE: Multiply the following pairs of numbers and check your answers in the back of the study unit.

1.	2.	3.	4.
46.37	21.3	307.	4.2
18.48	2.004	60.42	.00005

Work Unit 2-12. DIVISION OF DECIMALS

GIVEN TWO DECIMALS, APPLY THE OPERATIONS FOR DIVISION OF DECIMALS TO FIND THE QUOTIENT.

Division. Four things will be discussed: general division of decimals, division by powers of ten, reducing a common fraction to a decimal fraction, and rounding off.

General. Here, as in the other descussions, the purpose is not to teach you an operation, but to give you some insight into why the decimal point is moved, or not moved, in dividing decimals. You learned in the previous work unit that multiplication of a decimal by a power of ten has the effect of moving the decimal to the right. This in effect is what is done in division of decimals. If the divisor can be transformed into a whole number, the division can be accomplished in the same manner as division of integers. Two things are done without actually naming them as such. First, the divisor is actually being multiplied by some power of ten when the decimal is moved to the right to make it a whole number. Second, since the divisor and dividend can be considered as the numerator and denominator of a fraction, if the denominator (divisor) is multiplied by some number, then the numerator (dividend) must be multiplied by the same number in order not to change the value of the fraction. For example:

$$3.6\overline{)45.75}$$

In order to make 3.6 a whole number, the decimal point is moved one place to the right which is the effect of multiplying by 10. The same thing must be done to the dividend.

$$36\overline{)457.5}$$

Various methods are used to show that the decimal point has been moved. Two of the more common are illustrated.

$$.153\overline{)7.62}$$

$$.153.\overline{)7.620.}$$

Note that a zero was added to the dividend--the effect of multiplying by 1000.

$$.093\overline{).0176}$$

$$.093.\overline{).01\,.6}$$

You may remember this method of inserting a carat at the new location of the decimal point.

Remember also that the decimal point in the quotient (answer) will be directly above the new location of the decimal point in the dividend. The reason for this can be shown several ways. One is to recall the rules for multiplication of decimals. For example in this problem:

$$2.9.\overline{)\begin{array}{r}.186\\ .5394\end{array}}$$

in order to check the answer you must multiply the quotient times the divisor. This should equal the dividend. In this case .186 x 29 = 5.394. By working several examples, you can prove to yourself that the decimal point truly ends up in the correct place when it is placed directly over the one in the dividend.

Powers of ten. Division by a power of ten is the inverse of multiplying by powers of 10. Where multiplyinghad an increasing or upgrading effect, division has a decreasing or downgrading effect. Consequently division will cause the decimal point to "move" in the opposite direction or to the left. Note several examples.

$217{,}000 \div 1000 = 217$

$32{,}000 \div 10 = 3200$

$586.37 \div 100 = 5.8637$

$19.79 \div 1000 = .01979$

The generalization for division by a power of ten is that it has the effect of moving the decimal point in the dividend a number of places to the left equal to the number of zeros in the divisor.

Reducing a common fraction to a decimal fraction. Recall that earlier in the study unit it was mentioned that there was another way of reducing a common fraction to a decimal fraction. As mentioned, this method involved division of decimals. Now that this has been discussed, this much-used method may be covered. Since a common fraction is actually a division of the numerator by the denominator, this is what is done:

$\frac{1}{2} = 2\overline{)1.0}$ → .5; 1.0

$\frac{3}{4} = 4\overline{)3.00}$ → .75; 28; 20; 20

$\frac{3}{8} = 8\overline{)3.000}$ → .375; 24; 60; 56; 40; 40

$\frac{17}{20} = 20\overline{)17.00}$ → .85; 160; 100; 100

Each of these came out even (no remainder), but this is not always the case. In some instances, you may want to round off your answer to some particular decimal place.

Rounding off. Usually you are not lucky enough to have answers with no remainder as above. There will be a remainder to be expressed as a fraction. In division of decimals, this expression of the remainder may be eliminated by rounding off the answer to some desired decimal place. Common practice is to carry the division to one degree of accuracy more than is desired and then round off. For example, if the answer to a problem is 17.237 and accuracy to hundredths is all that is desired, the 7 may be eliminated. If the extra digit is 0, 1, 2, 3, or 4, it is usually dropped. If it is 5, 6, 7, 8, or 9, one is added to the digit to the left. Therefore, 17.237 rounded to hundredths would be 17.24. Some other examples are:

	Nearest hundredth	Nearest tenth
36.42793	36.43	36.4
427.6439	427.64	427.6
.34625	.35	.4
1.4963	1.50	1.5

EXERCISE: Divide the following items and express quotients rounded off to the nearest hundredths. Check your answers in the back of the study unit.

1. 97.42 ÷ 86.3

2. .49782 ÷ .031

3. 2.639 ÷ 2.2

Section V. PERCENTAGE

Work Unit 2-13. CONVERT FRACTIONS OR DECIMALS TO PERCENT

GIVEN A COMMON FRACTION OR A DECIMAL, APPLY THE OPERATIONS TO CONVERT TO PERCENT.

The word "percent" is derived from Latin. It was originally "per centum," which means "by the hundred." Thus the statement is often made that percent means hundredths.

Percentage deals with the group of decimal fractions whose denominators are 100--that is, fractions of two decimal places. Since hundredths were used so often, the decimal point was dropped and the symbol % was placed after the number and read "percent" (per 100). Thus, 0.15 and 15% represent the same value, 15/100. The first is read "15 hundredths," and the second is read "15 percent." Both mean 15 parts out of 100.

Changing decimals to percents. To change a decimal to a percent, the first step is to move the decimal point two places to the right. After moving the decimal point two places to the right, add the percent sign (%).

Example: .35 = 35%

A decimal point is not shown with whole percents. For example, 63% does not have a decimal point shown. Mentally, however, a decimal point is placed to the right of the number three (63.%).

Examples: .83 = 83%

.26 = 26%

Fractional percents do have decimal points.

$$56\frac{1}{2}\% = 56.5\%$$

Changing fractions to percents. To change a fraction to a percent, first change the fraction to a decimal and then the decimal to a percent.

$$\frac{3}{8} = .375 = 37.5\%$$

$$\frac{3}{4} = .75 = 75\%$$

EXERCISE: Change each of the following items to percent. Check your answers in the back of the study unit.

1. .005 = _____%
2. .12 = _____%
3. 1.25 = _____%
4. $\frac{1}{4}$ = _____%
5. $\frac{1}{5}$ = _____%
6. $\frac{9}{40}$ = _____%

Work Unit 2-14. CONVERT PERCENT TO A FRACTION OR DECIMAL

GIVEN A PERCENT, APPLY THE OPERATIONS TO CONVERT TO A COMMON FRACTION OR DECIMAL.

Changing percents to decimals. Since we do not compute with numbers in the percent form, it is often necessary to change a percent back to its decimal form. The steps are just the opposite of that used in changing decimals to percent. You change a percent to a decimal by dropping the percent sign and moving the decimal point two places to the left. For example, 12.5% becomes 12.5 after dropping the percent sign. Move the decimal point two places to the left and 12.5 becomes .125.

Examples: 34% = .34

24.5% = .245

125% = 1.25

When a fractional percent ($\frac{3}{4}$ %, $\frac{2}{34}$ %, $\frac{9}{16}$ %, etc.) is to be changed to a decimal, you must first change the fractional percent to its decimal equivalent percent (.75%, .059%, .56%) before you drop the percent sign and then move the decimal point two places to the left (.0075, .00059, .0056).

Examples: $5\frac{3}{4}\% = 5.75\% = .0575$

$\frac{1}{2}\% = .5\% = .005$

Changing percents to fractions. To change a percent to a common fraction, first change the percent to a decimal. The second step after changing the percent to a decimal is to change the decimal to a fraction and reduce to its lowest terms.

Examples: $80\% = .80 = \frac{80}{100} = \frac{4}{5}$

$65\% = .65 = \frac{65}{100} = \frac{13}{20}$

$22.5\% = .225 = \frac{225}{1000} = \frac{9}{40}$

EXERCISE: Change the following percents, first to decimals and then to fractions. Reduce the fractions to their lowest terms. Check your answers in the back of the study unit.

1. 40% = _____ = _____
2. 12.5% = _____ = _____
3. 32% = _____ = _____
4. 73% = _____ = _____

Work Unit 2-15. PERCENT OF ONE NUMBER TO ANOTHER NUMBER

GIVEN A NUMBER, USE THE OPERATIONS TO FIND WHAT PERCENT THIS NUMBER IS TO ANOTHER NUMBER.

You are now going to learn how to solve percentage problems. All percentage problems can be solved by substituting given information into the formula:

$$\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$$

Percentage numbers are all placed on one side of the equal sign in the formula and all other information on the other side.

Finding what percent one number is to another. For example, in the problem: There are 25 Marines in the class. Of the 25, 5 are NCO's. What percent of the Marines are NCO's? Substitute all known information into the formula.

Step #1 $\frac{\text{small}}{\text{large}} = \frac{\%}{100\%}$

Step #2 $\frac{5}{25} = \frac{x(\text{unknown})\%}{100\%}$

Step #3 $\frac{5}{25} = \frac{x}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $25x = 5$

Although this is actually an equation, and we will not be discussing algebra until the next study unit, we will cover enough algebra to complete the formula.

The Golden Rule. The multiplicative property states that if two equal numbers are multiplied by the same number, the products are equal. The division property states that if two equal numbers are divided by the same number, the quotients will be equal. Just as addition and subtraction are inverse operations, so are multiplication and division. One operation can undo the other. This rule also applies to solving equations involving the multiplication and division properties of equality.

$$25x = 5$$

You are looking for some number that when multiplied by 25 will give a product of 5. The objective in solving an equation is to isolate the variable, X in this case. Here we have a multiplication, so the inverse division operation will be applied.

Step #5 $\frac{25x}{25} = \frac{5}{25}$ (Note that $\frac{25x}{25} = 1x$. It is customary to drop the 1 since x and 1x are the same).

Step #6 $x = .20$

$x = 20\%$

Example: Of the 60 meals picked up by the right guide, 12 of them were turkey loaf. What percent were turkey loaf?

Step #1 $\frac{\text{small}}{\text{large}} = \frac{x\%}{100}$

Step #2 $\frac{12}{60} = \frac{x\%}{100\%}$ (Put known information into the formula).

Step #3 $\frac{12}{60} = \frac{x}{1.00}$ (Change percents to decimals and cross multiply).

Step #4 $60x = 12$

Step #5 $\frac{60x}{60} = \frac{12}{60}$

Step #6 $x = .20$ (Change decimal to percent).

$x = 20\%$

EXERCISE: Use the formula to solve the following problems by finding what percent one number is of another number. Check your answers in the back of the study unit.

1. The duty clerk made 14 errors in typing a memo of 60 words. What is his percent of error? Let x be the unknown.

2. The 3rd platoon has 4 people in sickbay. There are 32 people in the platoon. What percent of are in sickbay? Let x be the unknown.

Work Unit 2-16. FIND THE NUMBER THAT A PERCENT REPRESENTS

GIVEN A PERCENT OF A KNOWN NUMBER, USE THE PROPER OPEATIONS TO FIND THE NUMBER THIS PERCENT REPRESENTS.

Finding a percent of a number. This is probably the most used of all percentage problems. A typical problem would be: Twenty men work in the office. Ten percent can go on leave. What is the number of men that can go on leave? Here, as in the preceding work unit, the known information can be put into the formula:

Step #1 $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{x\text{(unknown)}}{20} = \frac{10\%}{100\%}$

Step #3 $\frac{x}{20} = \frac{.10}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $1x = 2.0$

Step #5 $x = 2$ men can go on leave

One more example may be helpful.

During the year (365 days), elements of the 1st platoon were on patrol 48% of the time. How many days were they on patrol?

Step #1 $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{x\text{(unknown)}}{365} = \frac{48\%}{100\%}$

Step #3 $\frac{x}{365} = \frac{.48}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $1x = 175.2$

Step #5 $x = 175.2$ days on patrol

EXERCISE: Use the formula to solve the following problems by finding the number that a percent represents. Check your answers in the back of the study unit.

1. Of 40 people in the platoon, 80% are qualified swimmers. How many are qualified swimmers? Let x be the unknown.

2. During the month of April (30 days), it rained 23% of the time. How many days dit it rain? Let x be the unknown.

Work Unit 2-17. FIND THE UNKNOWN NUMBER

GIVEN A PERCENT AND THE NUMBER IT REPRESENTS, USE THE OPERATIONS FOR FINDING THE UNKNOWN NUMBER.

Finding the unknown number. A typical problem would be: Pvt BONER bought a shirt for $10.50. This was 35% of the money he had saved for clothing. How much had he saved? As in the two preceding work units, the problem can be solved by putting the known information into the formula:

Step #1 $\frac{\text{sample number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{10.50}{x\text{(unknown)}} = \frac{35\%}{100\%}$ (Change percents to decimals and cross multiply)

Step #3 $\frac{10.50}{x} = \frac{.35}{1.00}$

Step #4 $.35x = 10.50$

Here as in work unit 2-15, you must use the inverse division operation. You are looking for some number that when multiplied by .35 will give you a product of 10.50.

Step #5 $\frac{.35x}{.35} = \frac{10.50}{.35}$

Step #6 $x = \$30$

One more example: Cpl SLICK spent $52 on liberty in Tokyo last Saturday. This was 65% of his bank roll. How much money did he have originally?

Step # 1 $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{52}{x\text{(unknown)}} = \frac{65\%}{100\%}$ (Change percents to decimals and cross multiply)

Step #3 $\frac{52}{x} = \frac{.65}{1.00}$

Step #4 $.65x = 52$

Step #5 $\frac{.65x}{.65} = \frac{52}{.65}$

Step #6 $x = \$80$

EXERCISE: Use the formula to solve the following problems by finding the unknown number. Check your answers in the back of the study unit.

1. 10% of the platoon are on R&R in Hong Kong. If there are 4 men on R&R, how many men are in the platoon? Let x be the unknown.

2. $\frac{3}{4}$% of the questions were incorrect. There are 12 incorrect. How many questions are there? Let x be the unknown.

SUMMARY REVIEW

In the preceding work units you have been introduced to the principles used in reducing fractions. You have also used the proper operations for addition, subtraction, multiplication, and division of fractions. You can now convert common fractions to decimals and vice versa. You have found that the operations used with decimals are an easy process complicated only slightly by placing of the decimal point. You have also learned to apply the principles of fractions and decimals to percent. You have found that all percentage problems can be solved by substituting given information into a formula.

Answers to Study Unit 2 Exercises

Work Unit 2-1.

1. $\frac{36}{48} = \frac{3}{4}$
common factor of 12

2. $\frac{28}{56} = \frac{1}{2}$
common factor of 28

3. $\frac{78}{102} = \frac{13}{17}$
common factor of 6

4. $\frac{99}{144} = \frac{11}{16}$
common factor of 9

5. $\frac{186}{378} = \frac{31}{63}$
common factor of 6

6. $\frac{680}{2520} = \frac{17}{63}$
common factor of 40

Work Unit 2-2.

1. $\frac{4}{9} = \frac{12}{27}$
common factor of 3

2. $\frac{3}{4} = \frac{75}{100}$
common factor of 25

3. $\frac{7}{8} = \frac{210}{240}$
common factor of 30

4. $\frac{5}{7} = \frac{40}{56}$
common factor of 8

5. $\frac{5}{12} = \frac{55}{132}$
common factor of 11

6. $\frac{9}{37} = \frac{27}{111}$
common factor of 3

Work Unit 2-3.

1. $\frac{3}{8} = \frac{9}{24}$

$+ \frac{1}{3} = \frac{8}{24}$

$\frac{17}{24}$

2. $\frac{1}{9} = \frac{2}{18}$

$+ \frac{1}{6} = \frac{3}{18}$

$\frac{5}{18}$

3. $\frac{2}{7} = \frac{24}{84}$

$+ \frac{1}{12} = \frac{7}{84}$

$\frac{31}{84}$

4. $\frac{3}{8} = \frac{15}{40}$

$+ \frac{7}{20} = \frac{14}{40}$

$\frac{29}{40}$

5. $\frac{3}{8} = \frac{45}{120}$

$\frac{1}{12} = \frac{10}{120}$

$\frac{2}{15} = \frac{16}{120}$

$+ \frac{3}{20} = \frac{18}{120}$

$\frac{89}{120}$

6. $7\frac{7}{8} = 7\frac{21}{24}$

$2\frac{1}{3} = 2\frac{8}{24}$

$+ 3\frac{5}{6} = 3\frac{20}{24}$

$12\frac{49}{24} = 14\frac{1}{24}$

7. $\frac{1}{6} = \frac{12}{72}$

$\frac{1}{12} = \frac{6}{72}$

$\frac{1}{9} = \frac{8}{72}$

$+ \quad \frac{1}{8} = \frac{9}{72}$

$\frac{35}{72}$

8. $3\frac{3}{7} = 3\frac{27}{63}$

$4\frac{11}{63} = 4\frac{11}{63}$

$+ \quad 10\frac{4}{9} = 10\frac{28}{63}$

$17\frac{66}{63} = 18\frac{3}{63} = 18\frac{1}{21}$

Work Unit 2-4.

1. $\frac{7}{9} = \frac{14}{18}$

$- \quad \frac{1}{6} = \frac{3}{18}$

$\frac{11}{18}$

2. $3\frac{1}{3} = 2\frac{8}{6}$

$- \quad \frac{5}{6} = \frac{5}{6}$

$2\frac{3}{6} = 2\frac{1}{2}$

3. $2\frac{3}{8} = 1\frac{11}{8}$

$- \quad \frac{5}{8} = \frac{5}{8}$

$1\frac{6}{8} = 1\frac{3}{4}$

4. $3\frac{5}{36} = 2\frac{41}{36}$

$- \quad 1\frac{4}{9} = 1\frac{16}{36}$

$1\frac{25}{36}$

5. $7\frac{5}{12} = 6\frac{17}{12}$

$- \quad 2\frac{7}{12} = 2\frac{7}{12}$

$4\frac{10}{12} = 4\frac{5}{6}$

6. $3\frac{2}{3} = 2\frac{20}{12}$

$- \quad \frac{11}{12} = \frac{11}{12}$

$2\frac{9}{12} = 2\frac{3}{4}$

7. $5 = 4\frac{3}{3}$

$- \quad 2\frac{2}{3} = 2\frac{2}{3}$

$2\frac{1}{3}$

8. $2 = 1\frac{64}{64}$

$- \quad \frac{49}{64} = \frac{49}{64}$

$1\frac{15}{64}$

Work Unit 2-5.

1. $\frac{5}{8} \times 16 = \frac{80}{8} = 10$

2. $\frac{4}{5} \times 2 = \frac{8}{5} = 1\frac{3}{5}$

3. $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$

4. $6\frac{2}{3} \times 15 = \frac{20}{3} \times 15 = \frac{300}{3} = 100$

5. $\frac{8}{9} \times 10 = \frac{80}{9} = 8\frac{8}{9}$

6. $8\frac{2}{3} \times 1\frac{1}{2} = \frac{26}{3} \times \frac{3}{2} = \frac{78}{6} = 13$

7. $8 \times 2\frac{1}{2} = 8 \times \frac{5}{2} = \frac{40}{2} = 20$

8. $7\frac{1}{2} \times \frac{4}{15} = \frac{15}{2} \times \frac{4}{15} = \frac{60}{30} = 2$

9. $\frac{15}{16} \times \frac{12}{25} = \frac{180}{400} = \frac{9}{20}$

10. $5 \times 17\frac{2}{3} = 5 \times \frac{53}{3} = \frac{265}{3} = 88\frac{1}{3}$

Work Unit 2-6.

1. $\frac{3}{8} \div \frac{3}{4} = \frac{\cancel{3}^{1}}{8} \times \frac{4}{\cancel{3}_{1}} = \frac{4}{8} = \frac{1}{2}$

2. $\frac{3}{7} \div \frac{5}{9} = \frac{3}{7} \times \frac{9}{5} = \frac{27}{35}$

3. $\frac{8}{17} \div \frac{3}{5} = \frac{8}{17} \times \frac{5}{3} = \frac{40}{51}$

4. $\frac{6}{27} \div \frac{12}{35} = \frac{\cancel{6}^{1}}{27} \times \frac{35}{\cancel{12}_{2}} = \frac{35}{54}$

5. $\frac{\cancel{14}^{2}}{\cancel{27}_{3}} \div \frac{7}{9} = \frac{\cancel{14}^{2}}{\cancel{27}_{3}} \times \frac{\cancel{9}^{1}}{\cancel{7}_{1}} = \frac{2}{3}$

6. $\frac{1}{2} \div \frac{7}{6} = \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{6}^{3}}{7} = \frac{3}{7}$

7. $\frac{\cancel{63}^{9}}{\cancel{360}_{8}} \div \frac{28}{45} = \frac{\cancel{63}^{9}}{\cancel{360}_{8}} \times \frac{\cancel{45}^{1}}{\cancel{28}_{4}} = \frac{9}{32}$

8. $6\frac{2}{3} \div 2\frac{3}{4} = \frac{20}{3} \div \frac{11}{4} = \frac{20}{3} \times \frac{4}{11} = \frac{80}{33} = 2\frac{14}{33}$

9. $114\frac{7}{12} \div 6\frac{7}{8} = \frac{1375}{12} \div \frac{55}{8} = \frac{\cancel{1375}^{25}}{\cancel{12}_{3}} \times \frac{\cancel{8}^{2}}{\cancel{55}_{1}} = \frac{50}{3} = 16\frac{2}{3}$

10. $\left(\frac{3}{7} - \frac{1}{14}\right) \div \frac{35}{49} = \left(\frac{6}{14} - \frac{1}{14}\right) \div \frac{35}{49} = \frac{5}{14} \div \frac{35}{49} = \frac{\cancel{5}^{1}}{\cancel{14}_{2}} \times \frac{\cancel{49}^{7}}{\cancel{35}_{7}} = \frac{7}{14} = \frac{1}{2}$

Work Unit 2-7.

1. $.06 = \frac{6}{100} = \frac{3}{50}$

2. $.278 = \frac{278}{1000} = \frac{139}{500}$

3. $.0075 = \frac{75}{10,000} = \frac{3}{400}$

4. $.432 = \frac{432}{1000} = \frac{54}{125}$

5. $.00375 = \frac{375}{100,000} = \frac{3}{800}$

6. $.8625 = \frac{8625}{10,000} = \frac{69}{80}$

Work Unit 2-8.

1. $\frac{13}{50} = \frac{26}{100} = .26$

2. $\frac{8}{25} = \frac{32}{100} = .32$

3. $\frac{74}{125} = \frac{592}{1000} = .592$

4. $\frac{7}{20} = \frac{35}{100} = .35$

Work Unit 2-9.

1.
```
  3.68
  4.975
  1.3
 16.42
------
26.375
```

2.
```
 64.2
   .04
 18.
------
17.37
------
99.61
```

3.
```
  8.637
492.
   .003
    .1
-------
500.740
```

Work Unit 2-10.

1.
```
 46.37
-18.48
------
 27.89
```

2.
```
 21.300
 - 2.004
--------
 19.296
```

3.
```
 307.00
 - 60.42
--------
 246.58
```

4.
```
 4.20000
 - .00005
---------
 4.19995
```

Work Unit 2-11.

1.
```
 46.37
 18.48
------
370 96
1854 8
37096
4637
--------
856.9176
```

2.
```
 21.3
  2.004
  -----
    852
    000
   000
  426
  -----
 42.6852
```

3.
```
  307.
   60.42
  ------
   6 14
  122 8
  000
 1842
 ------
 18,548.94
```

4.
```
 4.2
  .00005
  ------
     210
      00
     00
    00
   00
  ------
 .000210
```

Work Unit 2-12.

1. $86.3\overline{)97.4.200}$ = 1.13

```
      1.128
  863)97420.0
      863
      1112
       863
       2490
       1726
        7640
        6904
```

2. $.031\overline{).497.820}$ = 16.06

```
     16.058
  031)497820
      31
      187
      186
        182
        155
         270
         248
```

3. $2.2\overline{)2.6.390}$ = 1.20

```
      1.199
  22)26390
     22
     43
     22
     219
     198
      210
      198
```

Work Unit 2-13.

1. .005 = .5%
2. .12 = 12%
3. 1.25 = 125%
4. $\frac{1}{4}$ = 25%
5. $\frac{1}{5}$ = 20%
6. $\frac{9}{40}$ = 22.5%

Work Unit 2-14.

1. 40% = .40 = $\frac{40}{100}$ = $\frac{2}{5}$
2. 12.5% = .125 = $\frac{125}{1000}$ = $\frac{1}{8}$
3. 32% = .32 = $\frac{32}{100}$ = $\frac{8}{25}$
4. 73% = .73 = $\frac{73}{100}$

Work Unit 2-15.

1. The duty clerk made 14 errors in typing a memo of 60 words. What is his percent of error?

$\frac{14}{60} = \frac{x\%}{100\%}$ (Substitute known information into the formula).

$\frac{14}{60} = \frac{x}{1.00}$ (Change percents to decimals and cross multiply).

$60x = 14$

$\frac{60x}{60} = \frac{14}{60}$ (Apply the inverse division operation).

$x = .233$ (Change decimal to percent).

$x = 23.3\%$ of error.

2. The 3d platoon has 4 people in sickbay. There are 32 people in the platoon. What percent of the platoon are in sickbay?

$$\frac{4}{32} = \frac{x\%}{100\%}$$

$$\frac{4}{32} = \frac{x}{1.00}$$

$$32x = 4$$

$$\frac{32x}{32} = \frac{4}{32}$$

$$x = .125$$

$x = 12.5\%$ are in sickbay.

Work Unit 2-16.

1. Of 40 people in the platoon, 80% are qualified swimmers. How many are qualified swimmers?

$$\frac{x}{40} = \frac{80\%}{100\%}$$

$$\frac{x}{40} = \frac{.80}{1.00}$$

$$1x = 32$$

$x = 32$ qualified swimmers.

2. During the month of April (30 days), it rained 23% of the time. How many days did it rain?

$$\frac{x}{30} = \frac{23\%}{100\%}$$

$$\frac{x}{30} = \frac{.23}{1.00}$$

$$1x = 6.90$$

$x = 6.9$ days of rain.

Work Unit 2-17.

1. 10% of the platoon are on R&R in Hong Kong. If there are 4 men on R&R, how many are in the platoon?

$$\frac{4}{x} = \frac{10\%}{100\%}$$

$$\frac{4}{x} = \frac{.10}{1.00}$$

$$.10x = 4$$

$$\frac{.10x}{.10} = \frac{4}{.10}$$

x = 40 men in the platoon

2. $\frac{3}{4}$% of the questions were incorrect. There are 12 incorrect. How many questions are there?

$$\frac{12}{x} = \frac{\frac{3}{4}\%}{100\%}$$

$$\frac{12}{x} = \frac{.0075}{1.00}$$

$$.0075x = 12$$

$$\frac{.0075x}{.0075} = \frac{12}{.0075}$$

x = 1600 questions.

STUDY UNIT 3

INTRODUCTION TO ALGEBRA

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO IDENTIFY AND EVALUATE ALGEBRAIC EXPRESSIONS. YOU WILL BE ABLE TO SOLVE SIMPLE EQUATIONS AND INEQUALITIES. YOU WILL BE ABLE TO APPLY EQUATIONS TO PROPORTION AND PERCENTAGE.

Unfortunately there is no anecdote that can be told about Charlie Caveman in this study unit. We have reached a point of sophistication in our study of number beyond Charlie's profoundness. He did not have the number sense (or the need) necessary to extend his arithmetic knowledge into algebra. Although primitive man did not use algebra, there is evidence that early civilized man solved problems similar to the ones that are being solved in junior high school classrooms today. For example, on the Rhind Papyrus on display in the British Museum, there are 80 problems written by an Egyptian priest named Ahmes around 4000 years ago (would you believe 3800?). They have the title, "Directions For Obtaining the Knowledge of All Dark Things." Examples of the problems are:

Aha, its whole, its seventh is nineteen.

Heap, its two-thirds, its one-half, and its three-sevenths added together becomes thirty-three. What is the quantity?

Even with the strange words aha and heap which are indications of, "There is an unknown number," you should see the resemblance to some of the problems being taught today. Whereas these problems were solved mainly by trial and error by the Egyptians, today we would readily use our knowledge of algebra to transform them into equations.

The beginnings of algebra as we know it date from around AD 830 when again the Arabs and Hindus became an important part of our number history. The Arabs used algebra in their study of astronomy and it is believed that much of it was borrowed from the Hindus. The word algebra is derived from part of the title of a book written by an Arabian mathematician named Al-Khorwarazmi. The book, "ilm al-jabr wa'l magabalah," became known throughout Europe as "al-jabr" which you can see is not far from algebra.

Many students' only recollections of algebra are "all those x's and y's" and problems similar to the ones on the Rhind Papyrus. The problems will be discussed later in the study unit on problem solving. Right now we want to get at all those x's and y's and see what they are and how they can be used.

Throughout your study of algebra you will be confronted with two kinds of numbers: The numerical symbols that you have been accustomed to and literal numbers, that is, numbers represented by letters of the alphabet. These literal numbers are usually the unknown quantity that we are to find. Since literal numbers can have any value and are not fixed, they are called variables; that is, the value of x, y, z, m, n, etc. can vary from problem to problem. On the other hand, 7 is 7, 6 is 6, 34 is 34, etc., no matter where they are used. The value of these number symbols (numerals) will stay constant, and for this reason they are called constants. When a product involves a variable, it is customary to omit the multiplication symbol x so as not to confuse it with the literal number x. Thus, 3 times m is written as 3m and a times b is written ab. Recall that the product of two constants is not written in this manner, but is expressed by using the symbols of grouping that were discussed earlier. For example, 6 x 4 is not written as 64 but as 6(4), or (6)(4). Another customary convention in algebra is to use the raised dot to signify multiplication (6 · 4, a · b).

If you replace the variable in the expression 6y with each element of the natural numbers you would naturally get a different number as the product each time. An expression that contains a variable is called a variable expression or an open expression since you have left open the decision as to what number to specify for the variable. You do not know what number 6y represents until you specify the value of y. Any variable expression or numerical expression is known as an algebraic expression.

Work Unit 3-1. ALGEBRAIC EXPRESSIONS

IDENTIFY A MONOMIAL TERM IN AN ALGEBRIAC EXPRESSION

IDENTIFY A LIKE TERM.

IDENTIFY A POLYNOMIAL TERM IN AN ALGEBRAIC EXPRESSION.

What do these three things have in common?

- x
- $173562ab^3c^2m^2p^4x^7y^5$
- $\frac{a}{6} - 14(3a+7) + 17a^2 + 22$

Even though there is a difference in size and appearance, each one represents some number. If the values of the variables were known, it would be possible to find the number that is being represented. The common point of the three things is that they are all algebraic expressions. An algebraic expression then is any combination of numerals, literal numbers, and symbols that represent some number. Expressions, as we have just illustrated, can take a variety of forms. Let us examine the nomenclature of an expression.

Nomenclature. An algebraic expression in its simplest form is called a term. This is an expression that has none of its parts separated by addition or subtraction signs. From this you should see that in the examples above, x and $173562ab^3c^2m^2p^4x^7y^5$ are both terms. An expression of only one term is called a monomial. It can appear in several forms and can have several parts of its own or it may be as simple as a literal number, n. By adding one numeral to this literal number, for example, 5n, we have added new terminology. Five and n are factors of some product which until we replace n with a constant can only be known as 5n. Five is known as the coefficient on n. The literal number n is actually a coefficient of 5 also, but it has become conventional to refer to the numerical part of the term as the coefficient. For example, in the term $176y^3$, 176 is the coefficient of y^3. This brings up another item of nomenclature that was introduced in study unit 1: exponent. This is the small raised number that shows the power of a number or how many times this number, called the base, is to be used as a factor. For example, y^7 is y to the seventh power and indicates that y is to be a factor 7 times or $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$. In the term $3x^4$, the 3 is the coefficient which is a factor of the term. The exponent 4 signified that x, not 3x, is to be raised to the fourth power or used as a factor 4 times. Broken down, the term would appear as $3 \cdot x \cdot x \cdot x \cdot x$. Care must be taken so as not to associate the exponent with the wrong base. In the example above, the base is the x. In this example, $(3x)^4$, which at first glance may seem the same, the base is 3x. The parentheses indicate that the entire quantity 3x is to be used as a factor 4 times as opposed to the single quantity x mentioned previously. This example broken down would appear as $3 \cdot x \cdot 3 \cdot x \cdot 3 \cdot x \cdot 3 \cdot x$ or rearranging by use of the commutative principle for multiplication, $3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$.

Property of likeness. Earlier in the course the property of likeness was used in operating with numbers. This same property applies to terms. As you saw above, terms can take many different forms and in order for them to be combined they must be similar or like terms. This means that the literal parts of the terms must be the same. For example, 15ab and 12765ab are similar. The same is true with $17abcdx^3$ and $abcdx^3$, $2y^4$ and $7y^4$, c^2 and $25c^2$. Two terms such as 8x and 3ab or x and x^2 are unlike and cannot be combined or simplified. Remember, every part of the literal portion of the number must be identical for the terms to be similar.
$3x + 5x = 8x$, $10ab^2c - 4ab^2c = 6ab^2c$, and $17y^3 + 6y^3 - 4y^3 = 19y^3$. If the expression contains like and unlike terms, only the like ones are combined: $8ab + 4x - 2ab = 6ab + 4x$, $10x^3 + x^2 + 5x^2 - x^3 + x = 9x^3 + 6x^2 + x$.

Multiplication and division. Whereas adding and subtracting can only be done with like terms, any two terms can be multiplied or divided. We will confine ourselves; however, to like terms. The important difference with multiplication and division is that the letters are affected as well as the numbers. Two important things to remember: when multiplying like terms, you add the exponents. When dividing like terms, you will have no letters in the quotient.

Multiplication example: Multiply 7ac times 4ac. Each part of the two terms is multiplied 4·7, a·a, and c·c. The product is $28a^2c^2$.

$5b^3 \cdot 3b^3 = 15b^6$

$6y^7 \cdot 8y^7 = 48y^{14}$

$4a \cdot a = 4a^2$

$2x \cdot 3x \cdot 4x = 24x^3$

$17x^2 \cdot 3x^2 = 51x^4$

Division example: Divide $10b^2$ by $5b^2$

$\frac{10b^2}{5b^2} = 2$ Here we divide 10 by 5 and b^2 by b^2. With like terms, division of the letters will always be 1, therefore the answer is simply the quotient of the coefficients.

$\frac{5x^2yz}{2x^2yz} = 2\,1/2$

$\frac{1abc}{2abc} = 1/2$

$\frac{44x^3}{4x^3} = 11$

$\frac{69abcmnp}{23abcmnp} = 3$

Polynomials. Anyone familiar with prefixes of the English language and possessing a fair amount of logic should see that if monomial means one term, then polynomial means many terms. A polynomial then is an algebraic expression with many terms, actually two or more terms. There are two types of polynomial that appear more often than others and consequently have particular titles: binomial and trinomial. A binomial is a two-termed expression and trinomial is a three-termed expression. Keep in mind though that they are both polynomials. The determining factor of how many terms there are in an expression is the number of addition and subtraction symbols. That is, there will be either a + or - between terms. Some examples:

$a \oplus b$	2 terms - binomial
$x \oplus 5y \oplus 26z$	3 terms - trinomial
$54x^3y \oplus \frac{17y^2}{15d} \ominus 982abc$	3 terms - trinomial
$\frac{3(6+5)}{x} \oplus 14x(17-2)$	2 terms - binomial (even though there is a + and a - within each term)
$19a \oplus 23b \oplus 15c \oplus 16d \ominus 14x$	5 terms - polynomial
$\frac{5s + 3r}{v + 2w}$	1 term - monomial (even though there are + and - signs in both numerator and denominator, it is one fraction. The numerator and denominator are each binomials)

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. An algebraic expression of only one term is called a _____.

2. Which one of the algebraic expressions below is a monomial term?

   a. $100s - 35s$
   b. $45x^3y$
   c. $3x + y + x_2 + y$
   d. $(xz)^3 - 1x^2$

3. $5x^2$ and $15x^2$ are similar or _____ terms.

4. Which pair of algebraic expressions below contain like terms?

a. $135abc^2$, $45\ abc$
b. xyz^3, $xy(z)^3$
c. $5a^2$, $(5a)^2$
d. $16xy$, $1xy$

5. An algebriac expression with two or more terms is a _____ term.

6. Which of the below algebraic expressions is an polynomial term.

a. $\frac{6x + z}{x\ z}$
b. $46abc$
c. $\frac{37\ x\ y}{1x - y}$
d. $2x - x + 4z^3$

GIVEN AN ALGEBRAIC EXPRESSION, APPLY THE PROPER OPERATIONS TO EVALUATE IT.

Evaluation. Any algebraic expression can be evaluated to find what number it represents if the values of the variables are known. It is a matter of arithmetic and following the rules of order of operations that were presented earlier. Ability to evaluate expressions will be an aid to you in solving formulas and checking equations. Try to evaluate expressions in three steps: write the expression, substitute the given values, then do the arithmetic.

Examples:

If $p = 3$, evaluate $4p^2$

$4p^2$

$4(3)^2$ Note that the 3 is placed in parentheses to indicate multiplication by 4, and that the 3 was squared before being multipled by 4.

$4(9)$

36

If $m = 2$, evaluate $(8m)^2$

$(8m)^2$ Note that the entire quantity 8m is to be squared.

$(8 \cdot 2)^2$

$(16)^2$

256

If $x = 3$, evaluate $2x^2 + 4x + 5$

$2x^2 + 4x + 5$ Although some of the steps can be combined and others done mentally, it is suggested that beginners write each step as shown in the examples.

$2(3)^2 + 4(3) + 5$

$2(9) + 12 + 5$

35

If $x = 5$, $y = 2$, $z = 3$, evaluate $\frac{7y^2z - 2x^2}{xy}$

$$\frac{7y^2z - 2x^2}{xy}$$

$$\frac{7(2^2)3 - 2(5^2)}{5(2)}$$

$$\frac{7(4)3 - 2(25)}{10}$$

$$\frac{84 - 50}{10}$$

$$3\ 2/5$$

EXERCISE: Evaluate the expressions and check your answers in the back of the study unit.

7. $8r^2$ $(r = 5)$

8. $(8y)^2$ $(y = 5)$

9. $5y^2 - 3y + 4$ $(y = 1)$

10. $a^3 - 2a^2 + a + 4$ $(a = 5)$

11. $x^2 - y^2 + z^2$ $(x = 5,\ y = 2,\ z = 3)$

12. $\frac{x^2 + z^2}{y^2}$ $(x = 5,\ y = 2,\ z = 3)$

Work Unit 3-2. ALGEBRAIC SENTENCES

DETERMINE IF AN EQUATION IS TRUE OR FALSE

An algebraic sentence is a mathematical statement composed of algebraic expressions and one of these symbols: $=, \neq, >, <, \geq, \leq$. The first one, as you know, is the equal sign. The others, as you may or may not know, are, respectively: not equal, greater than, less than, equal to or greater than, and equal to or less than. Any sentence that uses the equal sign is an equation. Sentences that use the other signs are called inequalities.

Simple equations. As implied by the statement above, an equation is a statement that two things are equal. It may or may not be true. Thus, $4 + 5 = 9$ and $6 + 7 = 10 + 2$ are equations. The first is true because 4 +5 and 9 are symbols for the same number, but the second is false since 6 + 7 and 10 + 2 do not describe the same number. Again, statements such as $4 + 5 = 9$ and $6+7 = 10 + 2$, whether true or false, are called equations. Everything to the left of the equal sign is called the left member of the equation; all to the right of the equal sign is the right member. The solution or answer to an equation is called the root of the equation.

Before getting into equations containing variables let us use some arithmetic ability to determine whether some equations are true or false.

Example: Is this equation true or false? $9 - (1 + 8) = (9 - 1) + 8$

$9 - (1 + 8) = (9 - 1) + 8$
$9 - 9 = 8 + 8$
$0 = 16$

Obviously 0 does not equal 16. The equation is false.

True or false?

$4(9 - 5) - \frac{18 + 12}{5} = 4 + 3 \cdot 2$ Remember to multiply before doing addition.

$4(4) - \frac{30}{5} = 4 + 6$

$16 - 6 = 10$

$10 = 10$

True

Exercise: Are the following equations true or false? Check your answers in the back of the study unit.

1. $3 + 7 = 5 \times 2$
2. $4(5 + 1) = 4 \times 5 + 4 \times 1$
3. $4 \times 6\frac{1}{2} = 6 \times 4\frac{1}{2}$
4. $\frac{1}{2} \cdot 5(6 - 4) = \frac{15 - 7}{2} + \frac{19 - 16}{3}$
5. $7^2 - 8 \cdot 5 + 11 = 4^3 - 8^2 + 5^2$

GIVEN AN EQUATION, USE THE PROPER OPERATIONS TO FIND THE CORRECT ROOT

You can solve an equations such as $x + 7 = 13$ by inspection, that is, simply by examining and instinctively knowing that the number to be added to 7 to give 13 is 6. This method can be used to solve many simple equations. It takes no method or manipulation; simply inspection and recognition of the combinations. Witness the fact that many elementary school children are today solving problems such as:

$5 + \square = 8$

$\Delta - 10 = 5$

$\square - 4 = 6$

$* + 2 = 12$

$8 - \Delta = 1$

These are actually equations. The variable has been replaced by a different symbol, but nevertheless the children are solving equations. Of course your mathematical background has made you realize that all equations cannot be solved in inspection; there must be a mathematical method.

Addition and subtraction properties of equality. Recall that in study unit 1 some of the properties of number were discussed. By taking the properties of equality that were discussed and adding the addition and subtraction properties of equality, we obtain a process for solving equations. Let's look at an illustration.

Two Marines receive equal salaries	PFC Hard $100.50	PFC Charger = $100.50
Each one gets promoted to LCpl; a raise of $21.30	$100.50 + $21.30	= $100.50 + $21.30
The salaries change, but LCpl Hard's is still equal to LCpl Charger's	$121.80	= $121.80

This example of the addition property of equality shows that if the same number is added to equal numbers, the sums are equal. Symbolically, if $a = b$, then $a + c = b + c$. You should see that the subtraction property of equality could be proved in the same way. Symbolically, if $a = b$, then $a - c = b - c$. Let's see how these properties can be used to solve equations.

Look at these expressions:

100 - 7	26 + 14
48 + 12	n - 6
72 - 30	x + 4
	y - 7

Now suppose that you want to perform some operation that will preserve the quantity on the left. For example:

100 - 7_____ = 100	26 + 14_____ = 26
48 + 12_____ = 48	n - 6_____ = n
72 - 30_____ = 72	y - 7_____ = y

What can be inserted in the blanks to produce the results on the right? In the first one, if 7 is subtracted from 100, what must be done to return to 100? The answer is to perform the opposite or inverse operation with the same number.

100 - 7 + 7 = 100

What must be done to the example n - 6 to preserve the n? We must do the opposite or add 6.

n - 6 + 6 = n

The others then would look like this:

48 + 12 - 12 = 48
72 - 30 + 30 = 72
26 + 14 - 14 = 26
x + 4 - 4 = x
y - 7 + 7 = y

We need one more item to enable us to solve equations using the addition and subtraction properties of equality. Remember the Golden Rule of fractions? There is a similar one for equations. If states that whatever operation is performed on one member of an equation must also be performed on the other member. Now let's use these principles to solve an equation.

x - 6 = 19

By first using the addition property we can eliminate the 6 from the left member and get the x by itself. This is the objective in solving any equation: to isolate the variable in one member of the equation. The left member of the equation now looks like this:

x - 6 + 6 =

Now we must use the Golden Rule. Since 6 was added to the left member, it must also be added to the right member.

x - 6 + 6 = 19 + 6

$\boxed{x = 25}$

This was a simple equation, one that could actually be solved by inspection, so you probably knew that the answer was 25. Let's check it to be sure since the check is part of solving an equation. It is actually the process of evaluating an expression.

Check: $x - 6 = 19$
$25 - 6 = 19$
$19 = 19$ ✓

The left member equals the right member so 25 is the correct root of the equation. Let us look at several more examples:

$$y + 17 = 45$$
$$y + 17 - 17 = 45 - 17$$
$$\boxed{y = 28}$$

Check: $y + 17 = 45$
$28 + 17 = 45$
$45 = 45$ ✓

$$\frac{2}{9} + c = \frac{11}{9}$$

$$\frac{2}{9} - \frac{2}{9} + c = \frac{11}{9} - \frac{2}{9}$$

$$c = \frac{9}{9}$$

$$\boxed{c = 1}$$

Check: $\frac{2}{9} + c = \frac{11}{9}$
$\frac{2}{9} + \frac{9}{9} = \frac{11}{9}$
$\frac{11}{9} = \frac{11}{9}$ ✓

Note that if this equation is confusing it can be rearranged to look like this:

$$\frac{2}{9} + c = \frac{11}{9}$$
$$c + \frac{2}{9} = \frac{11}{9}$$

Do you remember why this can be done? If you said the commutative law for addition, you are correct. Some students prefer to change the equation to this form before going to the next step.

<u>More examples:</u>

$$x - .8 = 1.1$$
$$x - .8 + .8 = 1.1 + .8$$
$$\boxed{x = 1.9}$$

Check: $1.9 - .8 = 1.1$
$1.1 = 1.1$ ✓

$$314 = a + 165$$
$$314 - 165 = a + 165 - 165$$
$$\boxed{149 = a}$$

Note that here the variable is in the right member, but it is solved in the same manner.

Check: $314 = 149 + 165$
$314 = 314$ ✓

After solving several equations of this type you may feel that it is not necessary to write in the step involving the addition and subtraction property. Fine, this can be done mentally. It is suggested, however, that if solving equations is new to you that you include the step until you consistently solve each equation correctly and have confidence in your ability to add and subtract "in your head."

Multiplication and division properties of equality. The properties are quite simple and to the point. The multiplicative property states that if two equal numbers are multiplied by the same number, the products are equal. Symbolically, if $a = b$, the $ac = bc$. The division property states that if two equal numbers are divided by the same number, the quotients will be equal. Symbolically, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$. Just as addition and subtraction are inverse operations, so are multiplication and division. One operation can undo the other. The Golden Rule also applies to solving equations involving the multiplication and division properties of equality.

Example: $6x = 84$

We are looking for some number that when multiplied by 6 will give a product of 84. (Perhaps you can do this by inspection even though it is beyond the primary multiplication combinations.) Also recall that the objective in solving an equation is to isolate the variable, x in this case. From the practice with addition and subtraction, you know that in each case the inverse operation was applied. Here we have a multiplication, so the inverse operation division will be applied.

$6x = 84$

$\frac{6x}{6} = \frac{84}{6}$

$\boxed{x = 14}$

Note that $\frac{6x}{6} = 1x$. It is customary to drop the coefficient if it is 1 since x and 1x are the same.

Check: $6(14) = 84$

$84 = 84$

Examples:

$13m = 169$

$\frac{13m}{13} = \frac{169}{13}$

$\boxed{m = 13}$

Check: $13(13) = 169$

$169 = 169$ ✓

$1.5n = 3$

$\frac{1.5n}{1.5} = \frac{3}{1.5}$

$\boxed{n = 2}$

Check: $1.5(2) = 3$

$3 = 3$ ✓

$.7 = .7m$

$\frac{.7}{.7} = \frac{.7m}{.7}$

$\boxed{1 = m}$

Check: $.7 = .7(1)$

$.7 = .7$ ✓

$\frac{2}{3} = 6c$

$\frac{2}{3} \div 6 = \frac{6c}{6}$

$\frac{2}{3} \times \frac{1}{6} = c$

$\boxed{\frac{1}{9} = c}$

Check: $\frac{2}{3} = 6\frac{1}{9}$

$\frac{2}{3} = \frac{6}{9}$

$\frac{2}{3} = \frac{2}{3}$ ✓

In an equation involving division, the multiplication property will be used to find the root. For example, in $\frac{n}{4} = 6$, we are looking for some number n that when divided by 4 equals 6.

$\frac{n}{4} = 6$

$4 \cdot \frac{n}{4} = 6 \cdot 4$

$\boxed{n = 24}$

Since n is divided by 4, multiply it by 4 in order to get 1 as the coefficient of n

Check: $\frac{n}{4} = 6$

$\frac{24}{4} = 6$

$6 = 6 \checkmark$

Examples: $\frac{x}{19} = 6$

$19 \cdot \frac{x}{19} = 6 \cdot 19$

$\boxed{x = 114}$

Check: $\frac{114}{19} = 6$

$6 = 6 \checkmark$

$\frac{y}{5} = .8$

$5 \cdot \frac{y}{5} = .8 \cdot 5$

$\boxed{y = 4}$

Check: $\frac{4}{5} = .8$

$.8 = .8 \checkmark$

$\frac{x}{12} = \frac{5}{3}$

$12 \cdot \frac{x}{12} = \frac{5}{3} \cdot 12$

$\boxed{x = 20}$

Check: $\frac{20}{12} = \frac{5}{3}$

$\frac{5}{3} = \frac{5}{3} \checkmark$

As with addition and subtraction, the intermediate step can be mental. You are encouraged to include it until you have confidence in yourself and consistently get correct answers.

Equations involving the four operative properties of equality. Often equations may involve a combination of properties such as addition and multiplication, addition and division subtraction and multiplication, etc. The principles of solving the equations will remain the same, only the number of steps will change. The sequence in which to proceed is based on the principle of isolating the variable. Let's look at several examples.

$8y + 4 = 52$

Here we have a multiplication (8y) and an addition (8y + 4). In order to isolate the variable (y), the first step would be to eliminate the 4

$8y + 4 = 52$

$8y + 4 - 4 = 52 - 4$

$8y = 48$

Now we have an equation of the type solved in the last exercise.

$8y = 48$

$\frac{8y}{8} = \frac{48}{8}$

$\boxed{y = 6}$

Check: $8y + 4 = 52$

$8(6) + 4 = 52$

$48 + 4 = 52$

$52 = 52 \checkmark$

Note: Always check the root in the original equation. Not, for example, in $8y = 48$

Examples:

$$9a - 9 = 27$$
$$9a - 9 + 9 = 27 + 9$$
$$9a = 36$$
$$\frac{9a}{9} = \frac{36}{9}$$
$$\boxed{a = 4}$$

Check: $9a - 9 = 27$
$9(4) - 9 = 27$
$36 - 9 = 27$
$27 = 27$ ✓

$$2p - 5 = 8.5$$
$$2p - 5 + 5 = 8.5 + 5$$
$$2p = 13.5$$
$$\frac{2p}{2} = \frac{13.5}{2}$$
$$\boxed{p = 6.75}$$

Check: $2p - 5 = 8.5$
$2(6.75) - 5 = 8.5$
$13.50 - 5 = 8.5$
$8.5 = 8.5$ ✓

$$7n - 3n = 40 + 16$$
$$4n = 56$$
$$\frac{4n}{4} = \frac{56}{4}$$
$$\boxed{n = 14}$$

Note that here like terms must be combined first.
Check: $7(14) - 3(14) = 40 + 16$
$98 - 42 = 56$
$56 = 56$ ✓

$$x + 4 + 2x + 5 + 4x = 51$$
$$7x + 9 = 51$$
$$7x + 9 - 9 = 51 - 9$$
$$7x = 42$$
$$\frac{7x}{7} = \frac{42}{7}$$
$$\boxed{x = 6}$$

Check: $6 + 4 + 2(6) + 5 + 4(6) = 51$
$6 + 4 + 12 + 5 + 24 = 51$
$51 = 51$ ✓

Exercise: Solve the following equations and check your answers in the back of the study unit.

6. $6x - 3 = 27$

7. $\frac{2x}{5} = 86$

8. $7h + 7.79 = 29$

9. $.5c + 8 = 23$

10. $67 = 5m - 13$

11. $\frac{3x}{5} - 3 = 12$

12. $3x + 5x = 34 - 10$

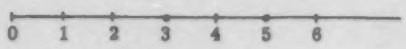
13. $45 = 8b + 7b + 30$

14. $13a + 2a + 5a + 85 = 95$

15. $8b + 2b + 7b + 187 = 221$

DETERMINE IF AN INEQUALITY IS TRUE OR FALSE

Inequalities. At the beginning of this work unit on algebraic sentences, you were given a group of symbols that denote inequalities. Ten years ago inequalities were discussed in college level math courses and sometimes in better high school classes Now this idea too, has invaded down to the elementary school level. Why? Basically to show comparisons between numbers. An elementary school child can readily interpret the statement $5 > 3$ as 5 is greater than 3 and know that this means that the number 5 is bigger or is more than the number 3. He also associates these numbers on the number line and knows that 5 is to the right of 3. The farther to the right, the larger the number.



At a more advanced level, inequalities may be applied to problems in economics, military tactics, industrial quality control, and research in general. Problems concerning the most profitable division of time on TV programs, the allocation of production facilities in a factory to achieve maximum profit, the determination of feed mixtures to minimize costs while satisfying standard food requirements, or the acceptance or rejection of manufactured items by use of sampling techniques represent a few of the many areas to which the mathematics of inequalities can be applied. Most of this is beyond the realm of Math For Marines, but some of the aspects of inequalities are practical, particularly to help you appreciate number relationships. Recall that an inequality is a statement that two quantities are not equal. Like an equation, it may or may not be true. For example, $12 \neq 8 + (5-3)$. Is this true or false?

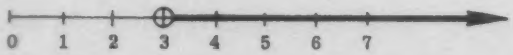
$12 \neq 8 + (5-3)$
$12 \neq 8 + 2$
$12 \neq 10$

This is true; 12 is not equal to 10. How about $11 < 8 + 2$ which is read 11 is less than $8 + 2$?

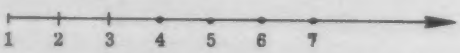
$11 < 8 + 2$
$11 < 10$

This is false, 11 is not less than 10.

The next step is to extend these concepts to expressions that include variables. For example, $x > 3$. What does this mean? If we are talking about the positive real numbers, it means all of the numbers to the right of 3 on the number line. Note that there is an open circle around the 3 to show that is is not part of the solution.



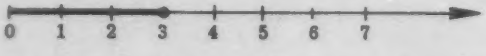
If we are talking about the natural numbers, it would be only the counting numbers to the right of 3



Some inequalities are a little more complicated and involve the same procedures as were used to solve equations. For example, $x + 2 \leq 5$.

$x + 2 \leq 5$
$x + 2 - 2 \leq 5 - 2$
$\boxed{x \leq 3}$

The graph would look like this for the positive real numbers. Note that the circle around 3 is solid this time to show that 3 is part of the solution.



This means that any number less than or equal to three is an answer to the inequality. In checking it 3 or any of the numbers less than 3 can be substituted for the variable.

$x + 2 \leq 5$
$2 + 2 \leq 5$
$4 \leq 5$ ✓

$x + 2 \leq 5$
$2\frac{9}{10} + 2 \leq 5$
$4\frac{9}{10} \leq 5$ ✓

Examples:

$$2x > 4$$
$$\frac{2x}{2} > \frac{4}{2}$$
$$\boxed{x > 2}$$

$$3x + 4 > 13$$
$$3x + 4 - 4 > 13 - 4$$
$$3x > 9$$
$$\frac{3x}{3} > \frac{9}{3}$$
$$\boxed{x > 3}$$

Exercise: Determine whether these inequalities are true or false: Check your answers in the back of the study unit.

16. $3 + 6 < 8 - 4$

17. $\frac{12}{4} > \frac{12}{3}$

18. $2^2 \neq 1^2(2)^2$

19. $4(9 - 5) - \frac{8 + 2}{5} \neq 4 + 3 \cdot 2$

20. $\frac{3}{5} + \frac{2}{5} < \frac{2}{5} + \frac{3}{5}$

21. $6 \cdot \frac{12 + 8}{5} > 2 \cdot 2$

Work Unit 3-3. PROPORTION AND PERCENTAGE

GIVEN A PROBLEM INVOLVING PROPORTIONS, SOLVE USING THE OPERATIONS FOR EQUATIONS.

Problems involving both proportion and percentage are readily solved with the use of equations. Although percentage has been discussed previously, it is a topic that has enough use to warrant its being covered again from a different aspect; that of solution by equation. Proportion is another type of equation that can be used to solve many practical problems.

Proportion. One day while taking ten from the hot sun, several members of the platoon were guessing at the height of the tree under which they were sitting. Pvt Buttplate stared at the shadows cast by various members of the platoon. He whipped out his tape measure and started measuring the length of the shadows. LCpl Flashhider, the machine gunner, had a shadow 42 in. long. His ammo carrier, Pfc A. Humper had a shadow 48 in. long. A small boy who had come to stare at the Marines had a shadow 20 in. long, while the tree had a shadow 40 ft long. Buttplate checked the height of the three persons whose shadows he had measured and scratched this table in the dirt:

	Shadow length	Height
Pfc Humper	48 in.	72 in.
LCpl Flashhider	42 in.	63 in.
Little boy	20 in.	30 in.
Tree	40 ft.	?

Buttplate thought for several minutes and then announced that the tree was 60 ft tall. Was he correct?

Let us see how Buttplate rationalized his answer. First he noticed that when he compared the shadow lengths to the actual heights they were all equal to the same thing:

$$\frac{20}{30} = \frac{2}{3}, \quad \frac{42}{63} = \frac{2}{3}, \quad \frac{48}{72} = \frac{2}{3}$$

This $\frac{2}{3}$ is called a ratio. It is the comparison of two things by division, and is read 2 to 3, the fraction line being "to." An older, but still used form is 2:3 with the colon being read "to." Ratios are reduced to lowest terms as indicated by all of the above being reduced to $\frac{2}{3}$.

Buttplate assumed (correctly) that the principle would hold true for all objects measured at the same time and place. He then has the problem $\frac{40}{?} = \frac{2}{3}$. He must find a fraction with a numerator of 40 that is equivalent to $\frac{2}{3}$. Since he had done study unit 2 of this course, he arrived at the correct answer, 60 ft. Just to check his theory, Buttplate decided to check another way. He found that Cpl Kbar was 66 in. tall. How long should his shadow be? $\frac{?}{66} = \frac{2}{3}$. Again using the principles that he learned in reducing fractions to equivalent forms, he found that Kbar's shadow should be 44 in. long. He measured it and found this to be correct.

Referring back at our example and the the definition of ratio, we see that a ratio compares just two numbers. In the example, we had two sets of numbers, one set from the lengths of shadows and one set from the heights. All of the ratios were equal to one another. They are said to be <u>proportional</u>. A <u>proportion</u> is a statement of equality of two ratios. Thus:

$$\frac{20}{30} = \frac{2}{3}, \quad \frac{42}{63} = \frac{2}{3}, \quad \frac{48}{72} = \frac{2}{3}, \quad \frac{40}{60} = \frac{2}{3}, \quad \frac{44}{66} = \frac{2}{3}$$

are all proportions. The equal sign is read "as". The first one then is read 20 is to 30 as 2 is to 3. Another way of writing a proportion is with a double colon for the "as." 20:30:: 2:3. Let us now see an important characteristic of proportions and how we can use the equation-solving techniques learned earlier to solve proportions.

$$\frac{2}{3} = \frac{6}{9} \quad \frac{3}{4} = \frac{24}{32} \quad \frac{5}{6} = \frac{15}{18} \quad \frac{3}{8} = \frac{375}{1000}$$

In each of these proportions, as in all proportions, there is an important relationship that exists between the two ratios.

For example:

$\frac{2}{3} = \frac{6}{9}$	$2 \cdot 9 = 3 \cdot 6$	$18 = 18$
$\frac{3}{4} = \frac{24}{32}$	$3 \cdot 32 = 4 \cdot 24$	$96 = 96$
$\frac{5}{6} = \frac{15}{18}$	$5 \cdot 18 = 6 \cdot 15$	$90 = 90$
$\frac{3}{8} = \frac{375}{1000}$	$3 \cdot 1000 = 8 \cdot 375$	$3000 = 3000$

If a proportion is true, the products of the numerator of one fraction and the denominator of the other fraction will be equal. This then can be useful in solving more difficult proportions where the outcome is not so obvious. For example, suppose Pvt Buttplate measured the shadow of another tree and found it to be 21 ft. How tall is the tree? We have then the length of the shadow (21) is to the height of the tree (x) as 2 is to 3.

$$\frac{21}{x} = \frac{2}{3}$$

From the characteristic just explained we know that the products of the numerator of one fraction and the denominator of the other will be equal.

$$\frac{21}{x} = \frac{2}{3}$$
$$2x = 21(3)$$
$$2x = 63$$

Now we have an equation of the form solved earlier in the work unit.

$$2x = 63$$
$$\frac{2x}{2} = \frac{63}{2}$$
$$x = 31\frac{1}{2}$$

The height of the tree is $31\frac{1}{2}$ ft.

Let us look at several more examples of solving proportions.

Example 1:

The truck convoy took 4 hours to travel the first 75 miles of the trip. Assuming the same average speed for the remaining 60 miles, how much longer will it take to reach the destination?

Actually the proportion can be set up in several ways. If we say that the first ratio is 4 hours to 75 miles, $\frac{4}{75}$, then the other would be unknown time, x, to 60 miles, $\frac{x}{60}$.

The proportion then is:

$$\frac{4}{75} = \frac{x}{60}$$
$$75x = 240$$
$$\frac{75x}{75} = \frac{240}{75}$$
$$x = 3.2 \text{ hr or 3 hours and 12 minutes}$$

We could do it another way by setting up ratios of same units. The ratio of unknown time to known time is $\frac{x}{4}$. The ratio of distance for unknown time to distance for known time is $\frac{60}{75}$.

The proportion is:

$$\frac{x}{4} = \frac{60}{75} \quad \begin{matrix}\text{Distance at unknown time} \\ \text{Distance at known time}\end{matrix}$$
$$75x = 240$$
$$x = 3.2$$

Example 2:

The engineers reported that the road up the mountain would have to have a slope of 6%, which means 6 ft of rise for every 100 ft of length. How much will the road rise in 1 mile?

Again the proportion can be set up several ways. One ratio is given 6 ft of rise per 100 ft of length or $\frac{6}{100}$. The other then would be an unknown rise, x, to the known distance 1 mile (5,280 feet) or $\frac{x}{5280}$. The proportion then is:

$$\frac{6}{100} = \frac{x}{5280}$$
$$100x = 31,680$$
$$\frac{100x}{100} = \frac{31,680}{100}$$
$$x = 316.8$$

The road rises 316.8 ft in 1 mile.

Another way to set up the problem would be the unknown rise, x, to the known rise, 6, or $\frac{x}{6}$ equals the distance of unknown rise, 5280, to the distance of the known rise or $\frac{5280}{100}$. The proportion would be:

$$\frac{x}{6} = \frac{5280}{100}$$
$$100x = 31,680$$
$$x = 316.8$$

Try this problem and then check your solution with the one presented below.

The messhall is making cookies to be served with the evening meal. Sgt Hashbrown's cookbook calls for 20 eggs in a recipe for 300 cookies. If he wanted to make 450 cookies, how many eggs would he use?

Solution:

$$\frac{20}{300} = \frac{x}{450}$$
$$300x = 9000$$
$$\frac{300x}{300} = \frac{9000}{300}$$
$$x = 30$$

30 eggs would be needed for 450 cookies.

Another solution:

$$\frac{\text{Unknown eggs } x}{20} = \frac{450 \text{ Cookies from unknown eggs}}{300 \text{ Cookies from known eggs}}$$

$$300x = 9000$$

$$x = 30$$

If you got this one correct, proceed to the exercise. If you got it wrong, go back and read again each of the examples presented. Then proceed to the exercise.

EXERCISE: Solve each proportion and check your answers in the back of the study unit.

1. $\frac{20}{8} = \frac{x}{6}$

2. $\frac{14}{30} = \frac{x}{90}$

3. $\frac{x}{42} = \frac{36}{27}$

4. $\frac{3}{x} = \frac{15}{75}$

Determine if the pairs of ratios are proportions

5. $\frac{6}{3}, \frac{16}{9}$

6. $\frac{48}{16}, \frac{42}{14}$

7. $\frac{15}{13}, \frac{45}{39}$

8. $\frac{68}{17}, \frac{76}{19}$

9. If bricks cost $40 per thousand, what is the cost of 8500 bricks?

10. If you are paid $135 for a job that requires 40 hours of work, how much would you expect to be paid for a job that requires 80 hours of work? (Assume the same rate of pay.)

GIVEN A PROBLEM INVOLVING PERCENTAGE , SOLVE USING THE OPERATIONS FOR EQUATIONS

Percentage. Earlier in the course, percentage was introduced and you were given a method to use for solving the various types of problems. Now that equation solving has been discussed we will see how this can be applied to percentage. There are three types of percentage problems, sometimes referred to as cases: Finding a percent of a number, finding what percent one number is of another, and finding a number when the percent of the number is known. It is a relatively simple matter to transform a sentence describing one of the percentage cases into an equation. The two key words in these sentences are "of" and "is." "Of" can be thought of as indicating multiplication and "is" can be replaced by an equal sign.

Finding a percent of a number. This is probably the most used of the three cases. A typical sentence would be: What is 35% of 14? Now let's take this and make an equation of it. "What" is the unknown (x), "is" will be replaced by the equal sign, the 35% is changed to a decimal, and "of" is replaced by a multiplication symbol.

What is 35% of 14?
$x = .35(14)$
$x = 4.9$
$\therefore$ 35% of 14 is 4.9

What is 86% of 741?
$x = .86(741)$
$x = 637.26$
$\therefore$ 86% of 741 is 637.26

Finding what percent one number is of another. A typical sentence would be: 20 is what percent of 80? Substitution of the symbols of the equation will be made in the same manner.

20 is what percent of 80?
$20 = x\%(80)$

Now the equation says that 20 is equal to some unknown percent times 80. One slight adjustment to the equation will put it in a more familiar form. By use of the commutative law for multiplication, change the positions of x% and 80.

$20 = 80x$ Note that the % sign has been dropped.

$\frac{20}{80} = \frac{80x}{80}$

$\frac{1}{4} = x$
$.25 = x$ } Both of these steps are not needed.
$25\% = x$

Check: $20 = 25\%(80)$
$20 = .25(80)$
$20 = 20$ ✓

45 is what percent of 120?

$45 = x\%(120)$
$45 = 120x$
$\frac{45}{120} = x$

$.375 = x$

$37\frac{1}{2}\% = x$

Check: $45 = 37\frac{1}{2}\%(120)$

$45 = .375(120)$
$45 = 45\checkmark$

Finding a number when a percent of the number is known. A typical sentence would be 40 is 2% of what number?

$40 = 2\%(x)$
$40 = .02x$
$\frac{40}{.02} = \frac{.02x}{.02}$
$2000 = x$

40 is 2% of 2000

Check: $40 = 2\%(2000)$
$40 = .02(2000)$
$40 = 40\checkmark$

120 is 80% of what number?

$120 = 80\%x$
$120 = .80x$
$\frac{120}{.80} = x$
$150 = x$

120 is 80% of 150

Check: $120 = 80\%(150)$
$120 = .80(150)$
$120 = 120\checkmark$

It is hoped that you have seen from this work unit how the use of equation-solving ability can greatly simplify the solution of percentage problems. Remember to always write the statement in words and then substitute the mathematical symbols for the words.

EXERCISE: Solve each percentage problem and check your answers in the back of the study unit.

11. What is $2\frac{1}{2}\%$ of 950?

12. What is 150% of 3248? (Note this is greater than 100%)

13. 7 is what % of 12?

14. 4.50 is 25% of what number?

15. 16 is what % of 48?

16. 80 is 40% of what number?

SUMMARY REVIEW

In the preceding work units you have been introduced to algebra. You have identified a monomial term, like term, and polynomial term in an algebriac expression. You can now apply the proper operations to evaluate an algebriac expression. You can determine if an equation or an inequality is true or false. When given an equation you can use the proper operations to find the correct root. You can now solve equations involving proportion and percentage.

Answers to Study Unit #3 Exercises

Work Unit 3-1.

1. monomial

2. b. $45x^3y$

3. like

4. d. 16xy, 1xy

5. polynomial

6. d. $2x - x + 4z^3$

7. $8r^2$ (r = 5)
$8(5)^2$
8(25)
200

8. $(8y)^2$ (y = 5)
$(8 \cdot 5)^2$
$(40)^2$
1600

9. $5y^2 - 3y + 4$ (y = 1)
$5(1)^2 - 3(1) + 4$
$5(1) - 3(1) + 4$
$5 - 3 + 4$
6

10. $a^3 - 2a^2 + a + 4$ (a = 5)
$5^3 - 2(5)^2 + 5 + 4$
$125 - 2(25) + 5 + 4$
$125 - 50 + 5 + 4$
84

11. $x^2 - y^2 + z^2$ (x=5, y=2, z=3)
$5^2 - 2^2 + 3^2$
$25 - 4 + 9$
30

12. $\frac{x^2 + z^2}{y^2}$ (x=5, y=2, z=3)
$\frac{5^2 + 3^2}{2^2}$
$\frac{25 + 9}{4}$
$\frac{34}{4}$
$8\frac{1}{2}$

Work Unit 3-2.

1. $3 + 7 = 5 \times 2$

$10 = 10$ True

2. $4(5 + 1) = 4 \times 5 + 4 \times 1$

$4(6) = 20 + 4$

$24 = 24$ True

3. $4 \times 6\frac{1}{2} = 6 \times 4\frac{1}{2}$

$26 = 27$ False

4. $\frac{1}{2} \cdot 5(6 - 4) = \frac{15 - 7}{2} + \frac{19 - 16}{3}$

$\frac{1}{2} \cdot 5(2) = \frac{8}{2} + \frac{3}{3}$

$\frac{1}{2} \cdot 10 = 4 + 1$

$5 = 5$ True

5. $7^2 - 8 \cdot 5 + 11 = 4^3 - 8^2 + 5^2$

$49 - 40 + 11 = 64 - 64 + 25$

$9 + 11 = 0 + 25$

$20 = 25$ False

6. $6x - 3 = 27$

$6x - 3 + 3 = 27 + 3$

$6x = 30$

$\frac{6x}{6} = \frac{30}{6}$

$x = 5$

7. $\frac{2x}{5} = 86$

$5 \cdot \frac{2x}{5} = 86 \cdot 5$

$2x = 430$

$\frac{2x}{2} = \frac{430}{2}$

$x = 215$

8. $7h + 7.79 = 29$

$7h + 7.79 - 7.79 = 29 - 7.79$

$7h = 21.21$

$\frac{7h}{7} = \frac{21.21}{7}$

$h = 3.03$

9. $.5c + 8 = 23$

$.5c + 8 - 8 = 23 - 8$

$.5c = 15$

$\frac{.5c}{.5} = \frac{15}{.5}$

$c = 30$

10. $67 = 5m - 13$

$67 + 13 = 5m - 13 + 13$

$80 = 5m$

$\frac{80}{5} = \frac{5m}{5}$

$16 = m$

11. $\frac{3x}{5} - 3 = 12$

$\frac{3x}{5} - 3 + 3 = 12 + 3$

$\frac{3x}{5} = 15$

$5 \cdot \frac{3x}{5} = 15 \cdot 5$

$3x = 75$

$\frac{3x}{3} = \frac{75}{3}$

$x = 25$

12. $3x + 5x = 34 - 10$

$8x = 24$

$\frac{8x}{8} = \frac{24}{8}$

$x = 3$

13. $45 = 8b + 7b + 30$

$45 = 15b + 30$

$45 - 30 = 15b + 30 - 30$

$15 = 15b$

$\frac{15}{15} = \frac{15b}{15}$

$1 = b$

14. $13a + 2a + 5a + 85 = 95$

$20a + 85 = 95$

$20a + 85 - 85 = 95 - 85$

$20a = 10$

$\frac{20a}{20} = \frac{10}{20}$

$a = \frac{1}{2}$

15. $8b + 2b + 7b + 187 = 221$

$17b + 187 = 221$

$17b + 187 - 187 = 221 - 187$

$17b = 34$

$\frac{17b}{17} = \frac{34}{17}$

$b = 2$

16. $3 + 8 < 8 - 4$

$9 < 4$ False

17. $\frac{12}{4} > \frac{12}{3}$

$3 > 4$ False

18. $2^2 \neq 1^2(2)^2$
$4 \neq 4$ False

19. $4(9-5) - \frac{8+2}{5} \neq 4 + 3 \cdot 2$
$4(4) - 2 \neq 4 + 6$
$16 - 2 \neq 10$
$14 \neq 10$ True

20. $\frac{3}{5} + \frac{2}{5} < \frac{2}{5} + \frac{3}{5}$
$1 < 1$ False

21. $6 \cdot \frac{12+8}{5} > 2 \cdot 2$
$6 \cdot 4 > 2 \cdot 2$
$24 > 4$ True

Work Unit 3-3.

1. $\frac{20}{8} = \frac{x}{6}$
$8x = 20(6)$
$8x = 120$
$\frac{8x}{8} = \frac{120}{8}$
$x = 15$

2. $\frac{14}{30} = \frac{x}{90}$
$30x = 14(90)$
$30x = 1260$
$\frac{30x}{30} = \frac{1260}{30}$
$x = 42$

3. $\frac{x}{42} = \frac{36}{27}$
$27x = 36(42)$
$27x = 1512$
$\frac{27x}{27} = \frac{1512}{27}$
$x = 56$

4. $\frac{3}{x} = \frac{15}{75}$
$15x = 3(75)$
$15x = 225$
$\frac{15x}{15} = \frac{225}{15}$
$x = 15$

5. $\frac{6}{3} = \frac{16}{9}$
$6 \cdot 9 = 3 \cdot 16$
$54 = 48$ No

6. $\frac{48}{16} = \frac{42}{14}$
$48 \cdot 14 = 16 \cdot 42$
$672 = 672$ Yes

7. $\frac{15}{13} = \frac{45}{39}$
$15 \cdot 39 = 13 \cdot 45$
$585 = 585$ Yes

8. $\frac{68}{17} = \frac{76}{19}$
$68 \cdot 19 = 17 \cdot 76$
$1292 = 1292$ Yes

9. $\frac{40}{1000} = \frac{x}{8500}$
$1000x = 340,000$
$\frac{1000x}{1000} = \frac{340,000}{1000}$
$x = \$340$

10. $\frac{135}{40} = \frac{x}{60}$
$40x = 8100$
$\frac{40x}{40} = \frac{8100}{40}$
$x = \$202.50$

11. $x = .025(950)$
$x = 23.75$
$2\frac{1}{2}\%$ of 950 is 23.75

12. $x = 1.5(3248)$
$x = 4872$
150% of 3248 is 4872

13. $7 = x\%\,(12)$
$7 = 12x$
$\frac{7}{12} = \frac{12x}{12}$
$58\frac{1}{3}\% = x$

14. $4.50 = 25\%(x)$
$4.50 = .25x$
$\frac{4.50}{.25} = \frac{.25x}{.25}$
$18 = x$
4.50 is 25% of 18

15. $16 = x\%(48)$
$16 = 48x$
$\frac{16}{48} = \frac{48x}{48}$
$33\frac{1}{3}\% = x$

16. $80 = 40\%(x)$
$80 = .40x$
$\frac{80}{.40} = \frac{.40x}{.40}$
$200 = x$
80 is 40% of 200

STUDY UNIT 4

UNITS OF MEASUREMENT

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO: CONVERT METRIC UNITS OF MEASUREMENT, USING THE DECIMAL SYSTEM OR POWERS OF TEN RATIOS. YOU WILL ALSO BE ABLE TO COMPUTE THE VARIOUS UNITS OF MEASURE USED IN THE U. S. AT THIS TIME: AND CONVERT METRIC UNITS TO U. S. UNITS OR VICE VERSA.

Company L will land across Red Beach One at 0700. The company strength is 173 men. How many LVT's will be needed to land the entire unit in one wave?

Kilo Company will land in LZ Hawk at 0715. There are twelve choppers available. Four CH-46's will be able to carry 4000 lb each, and eight UH-34's can carry 1600 lb each. Have your heliteam assignments ready in 2 hours.

How many yards are in a mile? How many meters in a mile? How many paces in a mile? These are figures that every Marine should know or know how to compute whether he is on a patrol, driving a truck in a convoy, working in an artillery FO team, or flying a plane.

Measurement came about primarily because of the need for surveying real estate and for measuring quantities in the market place. One of the early problems in measurement is one that still plagues us today. This is the units of measurement. The ancient cultures used convenient body measurements to express various lengths and widths, and a favorite cup, bucket, rock, or piece of iron to express weights and volume. Their problem was, as ours is, standardization. Using measures such as the width of the palm (the hand), the distance between the tip of the thumb and the tip of the little finger of a spread hand (the span), the distance from the tip of the elbow to the tip of the longest finger (the cubit), and the distance between the finger tips of each hand when the arms are outstretched (the fathom), you can see how the quantity of some substance would vary with the size of the merchant. The ancient yard which was the distance around a man's chest became half a fathom or the distance from the nose to the tip of an outstretched finger. You can see that three of Wilt Chamberlain's yards would be considerable different from three of Mickey Rooney's. During medieval times there were attempts made by various kings to standardize certain measures. Although this was heading in the right direction, the logic of the methods is dubious. For example, the land measurement called the rod is 16 1/2 ft. Here is the way it was decreed to be determined in the sixteenth-century.

If you stand at the door of a church on a Sunday, and have 16 men stop--tall ones and short ones--as they leave after service and have them stand so that their left feet are toe to heel behind each other, the length obtained shall be the right and lawful rod with which to measure and survey land. Furthermore, the sixteenth part of that distance shall be the right and lawful foot.

Section I. METRIC SYSTEM

Because of the general confusion with measuring systems that prevailed throughout the world, a conference of world scientists was called in France during the latter part of the eighteenth century. Based strictly on logic and science, this group developed the metric system which so nicely fits the decimal system of numeration. All measures are in a 1 to 10 ratio. The basic unit of length is the meter which was originally one ten-millionth of the distance from the equator to the north pole. Liquid measure is based on the liter which was difined as the volume of a cube with an edge dimension of one-tenth of a meter (a decimeter). The basic unit of weight is the gram which was defined as the weight of a cube of pure water at the temperature of melting ice, the cube being one-hundredth of a meter on an edge (a centimeter). Since 1799 the meter has been standardized to be the length of a platinum bar sealed in a case in the Archives of State in France. The standard of weight has been changed to the kilogram (1000 grams) and is represented by a platinum weight also in the Archives. With these basics defined, the rest of the metric system is simply based on ratios of powers of ten.

Work Unit 4-1. METRIC LINEAR UNITS

GIVEN A SERIES OF METRIC LINEAR UNITS, USE THE DECIMAL SYSTEM OR POWERS OF TEN TO CONVERT ONE METRIC UNIT TO ANOTHER METRIC UNIT.

GIVEN AN ARITHMETICAL PROBLEM INVOLVING VARIOUS LINEAR UNITS, COMPUTE USING THE DECIMAL SYSTEM TO CONVERT METRIC UNITS TO METRIC UNITS.

Metric length measurement. The basic metric unit for length is the meter. As previously mentioned, the length of the meter was at first determined as one ten-millionth part of the distance from the equator to the north pole. It was afterward found that there had been a slight error in its determination. At present the length of the meter is a bar made of 90 per cent platinum and 10 per cent iridium, called the international meter, and preserved at a temperature of $0^{\circ}C$ at the International Bureau of Weights and Measures near Paris, France. Two copies of this meter are in the United States, kept in the Bureau of Standards at Washington, D. C. One of these is used as the working standard and the other one for comparison. In order to insure still greater accuracy, these are compared at regular intervals with the international meter.

Tables and terms used. In the English system of measurement we have many different terms and numbers. These numbers bear no relation to one another. In the metric system we have only a few terms and but one single number, and that number is 10. Some of the most often used terms are listed in the following table:

Metric Linear Units

kilometer	(km)	= 1000	meters	=	10^3
hectometer	(hm)	= 100	meters	=	10^2
dekameter	(dam)	= 10	meters	=	10^1
meter	(m)	= 1	meter		(basic unit)
decimeter	(dm)	= 0.1	meter	=	10^{-1}
centimeter	(cm)	= 0.01	meter	=	10^{-2}
millimeter	(mm)	= 0.001	meter	=	10^{-3}

If the foregoing terms are carefully fixed in mind the table is easily formed. The prefixes kilo, hecto, and deka are attached to the word meter when the measurement is larger than a meter. If the measurement is smaller than a meter, the prefixes deci, centi, and milli are attached to the word meter. The expressions decimeter, dekameter, and hectometer are rarely used in actual work; the value of decimeters is usually expressed and centimeters, the values of dekameters and hectometers in meters or kilometers.

Operating within the metric system. The powers-of-ten ratios simplify working within the metric system. For example a distance of 7 meters 8 decimeters 4 centimeters can be written decimally as 7.84 meters. This can be added to another distance such as 1 meter 2 decimeters 3 centimeters (1.23) without any conversion.

7.84
1.23
9.07 meters or 9 meters 0 decimeters 7 centimeters.

Any of the four arithmetic operations can be done with the metric system by using the rules for decimals. Converting one unit of measure to another (which is actually either multiplication or division) is done by moving the decimal points in the same manner as was explained in study unit 2 on powers of ten. This is a process that is needed regularly for changing map distances to ground distances and vice versa. Our main concern will be with centimeters, meters, and kilometers and the changing of one to another.

To change centimeters to meters, move the decimal point two places to the left. (Note in the linear measure table that meters and centimeters are two places apart.)
Example: 125,000 centimeters = 1,250 meters.

To change meters to kilometers, move the decimal point three places to the left. (Note that meters and kilometers are three places apart in the table.)
Example: 1,250 meters = 1.25 kilometers.

To change centimeters to kilometers, move the decimal point five places to the left.
Example: 125,000 centimeters = 1.25 kilometers.

To change kilometers to meters, move the decimal point three places to the right. (Note that when changing from large units to smaller units, it may be necessary to add zeros.)
Example: 1.25 kilometers = 1,250 meters.

To change meters to centimeters, move the decimal point two places to the right.
Example: 1,250 meters = 125,000 centimeters.

To change kilometers to centimeters, move the decimal point five places to the right.
Example: 1.25 kilometers = 125,000 centimeters.

Let us take another distance and take it through from kilometers to centimeters.

11 kilometers = 110 hectometers*
= 1,100 dekameters*
= 11,000 meters
= 110,000 decimeters*
= 1,100,000 centimeters

* Not usually used. Inserted to show the progression.

Here is another example working in the opposite direction:

50,000 centimeters = 5000 decimeters
= 500 meters
= 50 dekameters
= 5 hectometers
= .5 kilometers

In summary, keep in mind the operations with powers of ten (try to associate them with the metric prefixes: centi, hecto, kilo, etc.) and follow the rules for working with decimals (remember that for changing larger units to smaller units, move the decimal point to the right and vice versa).

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. Change 43,000 centimeters to meters.
2. Change 15 kilometers to meters.
3. Change 600 meters to kilometers.
4. Change 7 kilometers to centimeters.
5. On a map with a scale of 1:50,000 (1 unit on the map equals 50,000 of the same units on the ground). The distance between point A and B is 18 centimeters. What is the actual ground distance in meters?
6. On a 1:25,000 map, 14.3 centimeters equals how many kilometers?

Work Unit 4-2. METRIC CAPACITY UNITS

GIVEN A SERIES OF METRIC CAPACITY UNITS, USE THE DECIMAL SYSTEM OR POWERS OF TEN TO CONVERT METRIC UNITS TO ANOTHER METRIC UNIT.

GIVEN AN ARITHMETICAL PROBLEM INVOLVING VARIOUS METRIC CAPACITY UNITS, COMPUTE USING THE DECIMAL SYSTEM TO CONVERT METRIC UNITS TO METRIC UNITS.

Metric capacity. The basic metric unit for capacity (fluid) is the liter. The liter is determined by the measurement of a cubic decimeter (a cube one-tenth of a meter on each side).

Capacity table and terms. The same prefixes that were discussed in work unit 4-1 will also apply when working with the basic metric unit for capacity, the liter. The prefixes kilo, hecto, and deka are attached to the word liter when the measurement is larger than a liter. The prefixes deci, centi, and milli are attached to the word liter if the measurement is smaller than the liter.

Metric Capacity Units

kiloliter	(kl)	=	1000 liters	=	10^3
hectoliter	(hl)	=	100 liters	=	10^2
dekaliter	(dal)	=	10 liters	=	10^1
liter	(l)	=	1 liter		(basic unit)
deciliter	(dl)	=	0.1 liter	=	10^{-1}
centiliter	(cl)	=	0.01 liter	=	10^{-2}
milliliter	(ml)	=	0.001 liter	=	10^{-3}

The expressions or terms kiloliter, dekaliter, deciliter, and centiliter are rarely used in actual day-to-day work. The value of kiloliter is usually expressed in hectoliters, the value of dekaliters in liters, the values of deciliters and centiliters in liters or milliliters. Our main concern will be with hectoliters, liters, and milliliters and the changing of one to another.

Operating within the metric capacity units. The same operations used in work unit 4-1 also apply when working with metric capacity units. Any of the four arithmetic operations can be done using the rules for decimals or the powers-of-ten ratios. Converting one unit to another unit is done by moving the decimal points.

To change liters to milliliters, move the decimal point three places to the right. (Note in the capacity table that liters and milliliters are three places apart).
Example: 6 liters = 6,000 milliliters.

To change liters to hectoliters, move the decimal point two places to the left. (Note that liters and hectoliters are two places apart in the table.)
Example: 450 liters = 4.5 hectoliters.

To change hectoliters to milliliters move the decimal point five places to the right.
Example: 1.25 hectoliters = 125,000 milliliters.

To change hectoliters to liters, move the decimal point two places to the right.
Example: 3.6 hectoliters = 360 liters.

To change milliliters to liters, move the decimal point three places to the left.
Example: 750 milliliters = .75 liters.

To change milliliters to hectoliters, move the decimal point five places to the left.
Example: 3,550 milliliters = .0355 hectoliters.

Let us take another capacity measurement and take it through from hectoliters to milliliters.

25 hectoliters = 250 dekaliters*
= 2,500 liters
= 25,000 deciliters*
= 250,000 centiliters*
= 2,500,000 milliliters

* Not usually used. Inserted to show the progression.

Keep in mind the powers of ten and try to associate them with the metric prefixes: kilo, hecto, centi, and etc.. Remember that for changing larger units to smaller units, move the decimal point to the right and vice versa.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. Change 155 liters to milliliters.

2. Change 755 liters to hectoliters.

3. Change 4,834 milliliters to liters.

4. Express the sum of 6.5 hectoliters and 350 milliliters in liters.

5. Find the difference between 4.75 liters and 750 milliliters.

6. Express the sum of 36 liters and 4,000 milliliters in hectoliters.

Work Unit 4-3. METRIC WEIGHT UNITS

GIVEN A SERIES OF METRIC WEIGHT UNITS, USE THE DECIMAL SYSTEM OR POWERS OF TEN TO CONVERT ONE METRIC UNIT TO ANOTHER METRIC UNIT.

GIVEN AN ARITHMETICAL PROBLEM INVOLVING VARIOUS METRIC WEIGHT UNITS, COMPUTE USING THE DECIMAL SYSTEM TO CONVERT METRIC UNITS TO METRIC UNITS.

Metric weight. The basic metric unit for weight is the gram. The gram is defined as the weight of a cube (one-hundredth of a meter) of pure water at the temperature of melting ice.

Weight table and terms. The prefixes discussed in work units 4-1 and 4-2 also apply to the metric weight unit gram. When the measurement is larger than the gram, the prefixes kilo, hecto, and deka are attached. If the measurement is smaller than the gram, the prefixes deci, centi, and milli are attached to the unit gram.

Metric Weight Units

Tonne (metric ton)	(T)	=	1,000,000 grams	=	10^6
kilogram	(kg)	=	1,000 grams	=	10^3
hectogram	(hg)	=	100 grams	=	10^2
dekagram	(dag)	=	10 grams	=	10^1
gram	(g)	=	1 gram	=	(basic unit)
decigram	(dg)	=	0.1 grams	=	10^{-1}
centigram	(cg)	=	0.01 gram	=	10^{-2}
milligram	(mg)	=	0.001 gram	=	10^{-3}

The term tonne (metric ton) or megagram is introduced in this work unit to show you the expression used when weighing large or bulky amounts. For measures of weight the terms gram, kilogram, and tonne are the only three usually employed in practice. The values of hectograms and dekagrams are normally expressed as kilograms. Decigrams, centigrams, and milligrams will usually be expressed in values of the gram. Our main concern in this work unit will be with grams, kilograms, and tonne (megagrams).

Operating within the metric weight units. The powers-of-ten or rules for decimals also apply when working within metric weight units. Changing from one unit to another unit is as easy as moving the decimal point. If the unit is larger than the gram, move the decimal to the left. When the unit is smaller than the gram, move the decimal point to the right.

To change grams to kilograms, move the decimal point three places to the left.
Example: 9,754 grams = 9.754 kilograms.

To change kilograms to tonne (megagrams), move the decimal point three places to the left.
Example: 11,300 kilograms = 11.3 tonne.

To change tonne to kilograms, move the decimal point three places to the right.
Example: 5.75 tonne = 5,750 kilograms.

Let us take another weight and take it through from tonne (megagrams) to grams.

.5 tonne = 500 kilograms
= 5,000 hectograms
= 50,000 dekagrams
= 500,000 grams

Here is another example working in the opposite direction.

1,550,000 grams = 155,000 dekagrams
= 15,500 hectograms
= 1,550 kilograms
= 1.55 tonne

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. Change 3,450 kilograms to tonne.
2. Change 500 grams to kilograms.
3. Change 430 tonne to kilograms.
4. Change 750,000 grams to tonne.
5. Find the difference between 167 kilograms and 60,000 grams.
6. Express the sum of 37 tonne and 650,000 grams in kilograms.

Section II. U. S. SYSTEM

All of our present day units have some historical background such as this one for the rod and foot. The background of measures of capacity and weight is even more confusing and it remains this way today. There is a variance between the U.S. gallon and the British gallon and to make it more confusing, there are difference in areas of the U.S.. For example, the National Bureau of Standards says that within the U.S. there are eight different weights for a ton, nine different volumes for a barrel, and a different bushel for charcoal, corn, and potatoes. Out of this chaotic situation, the Bureau of Standards has tried to make some order and there are some generally accepted measures used throughout the U. S.

Work Unit 4-4. U.S. LINEAR MEASURE

GIVEN A SERIES OF U.S. LINEAR MEASUREMENTS, COMPUTE USING LIKE COLUMNS TO LINE UP THE UNITS OF MEASURE.

Units of linear measure. Unlike the metric system where there are only a few terms and but one number to remember, the U.S. system of measuring length contains many terms and numbers. The numbers in the U.S. system bear no relation to one another and can cause a bit of confusion when trying to remember which number goes with which term.

Linear measure table. Some of the terms are included just for information while others are used often enough that they should be common knowledge and should be committed to memory.

Linear Measure

12	inches (in.)	= 1	foot (ft)
36	inches	= 1	yard (yd)
3	feet	= 1	yard (yd)
16 1/2	feet	= 1	rod (rd)
5 1/2	yards	= 1	rod (rd)
63,360	inches	= 1	mile (mi)
5,280	feet	= 1	mile (mi)
1,760	yards	= 1	mile (mi)
320	rods	= 1	mile (mi)

Expressing and computing. You can see from the table that there are, in some cases, several ways of expressing a measurement. The yard is 3 feet or 36 inches; the mile is 5,280 feet, 1,760 yards or 63,360 inches. Usually though, measurements do not come out even and must be expressed in fractional parts or in terms of a smaller unit. For example, 132 inches is 11 feet or 3 2/3 yards. Conversely, 3 2/3 yards can be expressed as 11 feet or 132 inches. The form of expressing any measurement is usually determined by the preference of the user except in cases where something must be ordered in standard packages or sizes or when a standard format is used. For example, in ordering a 2 x 4, 8 feet long you would not ask for one 2 2/3 yards long nor, would you give them the measurement in inches.

Computations with these units can be simplified if you relate them to our decimal system. You know that when we add a column of numbers we add like columns: units, tens, hundreds, etc. This in fact is what you have when you add feet and inches, for example. Whereas 26 is 2 tens and 6 units, 31 inches is 2 feet and 7 inches. Six thousand feet is 1 mile, 240 yards or 1 mile, 720 feet. To compute with the various measures, then, it is only necessary to line up the proper units.

Examples: What is the total length of two boards: one 3 feet, 7 inches and the other 4 feet 8 inches?

3 ft 7 in.
4 ft 8 in.
7 ft 15 in.

Since 15 inches is greater than 1 foot, the foot is added (or carried) to the foot column and the remainder of the 15 inches is expressed in inches.

7 ft, 15in. = 8ft, 3 in.

How many miles were run if the first squad ran 2,520 yards, the second squad ran 2,760 yards, and the third squad ran 2,640 yards?

Method 1:
2,520 yds
2,760 yds
2,640 yds
7,920 yds

7,920 yards = 4 miles, 880 yards or in miles only, 4 1/2 miles.

Method 2:

2,520 yards	=	1 mile,		760 yds
2,760 yards	=	1 mile,	1,000 yds	
2,640 yards	=	1 mile,		880 yds
		3 miles,	2,640 yds	

3 miles, 2,640 yards = 4 miles, 880 yards = 4 1/2 miles

In both methods division has been used, although it is not indicated. For example to change the 7,920 yards to miles we divided 7,920 yards by 1,760 to get miles. In metnod 2, we divided each distance by 1,760 and then added like columns. In most cases it is best to change everything to the same unit.

When multiplication is necessary to solve a problem, the various units can be changed as you go along or they can be changed at the end.

Example: Changing as you multiply.

Multiply 8 times 2yd, 2ft, 2in.

8 x 2 in. = 16 in. which is 1 ft, 4 in; carry 1 ft
8 x 2 ft. = 16 ft. + 1 ft = 17 ft which is 5 yd, 2 ft; carry 5 yd
8 x 2 yd = 16 yd + 5 yd = 21 yd

21 yd, 2 ft, 4 in.

EXERCISE: Solve the following items and check your answers in the back of the study unit.

1. Add 3ft 4 in., 27 in., 1 1/2 yd, 2 3/4ft.

2. Multiply 5yd, 2ft, 27in. by 19

3. Divide 23 yd, 1 ft by 10

Work Unit 4-5. U.S. CAPACITY MEASURE

GIVEN VARIOUS CAPACITY MEASUREMENTS, COMPUTE USING LIKE COLUMNS TO LINE UP THE UNITS OF MEASURE.

Units of capacity measure. Here, as in the preceding work unit, the capacity measure contains many terms and numbers. The term capacity measurement refers to many types of measurement such as liquid measure and dry measure. In this work unit we will be primarily concerned with liquid measure.

Capacity measure table. Most of the terms in the table are common knowledge and should be easily remembered.

Capacity Measure

8 fluid ounces (oz.)	=	1 cup		
16 fluid ounces	=	1 pint	(pt)	
2 cups	=	1 pint	(pt)	
32 fluid ounces	=	1 quart	(qt)	
2 pints	=	1 quart	(qt)	
4 quarts	=	1 gallon	(gal)	
128 fluid ounces	=	1 gallon	(gal)	
31 1/2 gallons	=	1 barrel		

Expressing and computing. Capacity measurements can be expressed in many different ways. The gallon is 4 quarts or 128 fluid ounces, the quart is 32 fluid ounces or 2 pints. As previously mentioned, measurements do not always come out even and must be expressed in fractional parts or in terms of a smaller unit. For example, 192 fluid ounces is 1 1/2 gallons or 1 gallon 2 quarts.

Computations with liquid capacity measurements are made in the same manner as with linear measurements. Like units are placed in their proper columns.

Examples: What is the total capacity of two containers; one 2 gallons, 3 quarts and the other 3 gallons, 2 quarts?

2 gallons 3 quarts
3 gallons 2 quarts
5 gallons 5 quarts

Since 5 quarts is greater than 1 gallon, the gallon is added (or carried) to the gallon column and the remainder of the 5 quarts is expressed in quarts.
5 gallons, 5 quarts = 6 gallons, 1 quart

How many gallons of milk were consumed if the first squad drank 20 pints, the second squad drank 18 pints, and the third squad drank 27 pints?

Method 1: 20 pints
18 pints
27 pints
65 pints

65 pints = 32 qt, 1 pt
32 qt = 8 gallons
65 pints = 8 gallons, 1 pt or in gallons only, 8 1/8 gallons

Method 2: 20 pints = 2 gal, 2 qt
18 pints = 2 gal, 1 qt
27 pints = 3 gal, 1 qt, 1 pt
= 7 gal, 4 qt, 1 pt = 8 gal, 1 pt

In both methods division has been used, although it is not indicated. For example, to change the 65 pints to quarts we know two facts: There are 2 pints in a quart and 8 pints in a gallon. We divided 65 by 2 to get quarts; we could have divided 65 by 8 to get gallons directly. In method 2, we know that 4 qt is 1 gal, which is actually dividing by 4 qt by 4 to give 1, which is then carried to the gallon column. There are many problems that use just the division step for solution. In most cases it is best to change everything to the same unit.

Multiplication with the capacity units is done the same as was discussed in work unit 4-4. The various units can be changed as you go along or they can be changed at the end.

Example: Changing as you multiply.

Multiply 8 times 3 gal, 2 qt, 1 pt

8 x 1 pt = 8 pints which is 4 quarts, carry 4 quarts
8 x 2 qt = 16 quarts + 4 quarts = 20 quarts which is 5 gallons, carry 5 gallons.
8 x 3 gal = 24 gal + 5 gal = 29 gallons

EXERCISE: Solve the following items and check your answers in the back of the study unit.

1. Add 3 gal, 3 qt, 1 pt
   5 gal, 2 qt
   6 gal, 1 qt, 1 pt

2. Multiply 2 gal, 3 qt, by 417

3. Divide 6 gal, 3 qt, 1 pt by 11

Work Unit 4-6. U.S. WEIGHT MEASURE

GIVEN VARIOUS WEIGHT MEASURMENTS, COMPUTE USING LIKE COLUMNS TO LINE UP THE UNITS OF MEASURE.

Units of weight measure. The weight of any object is simply the force or pull with which it is attracted toward the center of earth by gravitation. There are three common systems of weights in the U.S.: Troy weight, used in weighting jewelry; apothecaries' weight, used in weighting small amounts of drugs; and avoirdupois weight, used for all ordinary purposes. Our primarily concern in this work unit will be with avoirdupois weight.

Weight measurement table. The terms listed in the following table are quite common and should be committed to memory.

Weight Measure

*	437 1/2	grains (gr)	= 1 ounce (oz.)
*	7,000	grains	= 1 pound (lb)
	16	ounces	= 1 pound
	2,000	pounds	= 1 ton
	2,240	pounds	= 1 long ton

* A grain was originally a grain of wheat but is now described in relation to a cubic inch of distilled water.

Expressing and computing. Expressing weight measurements can be done in several different ways. The ton is 2,000 pounds, the pound is 16 ounces or 7,000 grains. Here, as with linear and capacity measurements, the measurements do not always come out even and must be expressed in fractional parts or in terms of a smaller unit. For example, 2,500 pounds is 1 1/4 tons or 1 ton 500 pounds.

Computations with the weight measurements are made in the same manner as with linear or capacity measurements. To add or subtract numbers, they must be lined up in like columns.

Examples: What is the total weight of two boxes, one 10 pounds, 14 ounces and the other 19 pounds, 6 ounces?

10 lb, 14 oz.
19 lb, 6 oz.
29 lb, 20 oz.

Since 20 oz. is greater than 1 lb, the pound is added to the pound column and the remainder of the 20 oz. is expressed in ounces.

29 lb, 20 oz. = 30 lb, 4 oz.

There are many problems where division will be needed to find the solution. The two methods discussed in work units 4-4 and 4-5 can be used or in most cases it may be best to change everything to the same unit. For example, how many bags of sugar, each weighing 1 lb, 6 oz., can you get from one large quantity weighing 6 lb, 14 oz?

1 lb, 6 oz $\overline{)6\text{ lb},\ 14\text{ oz}}$ = 22 oz $\overline{)110\text{ oz}}$ (5)

110 oz

5 bags

To solve problems using multiplication, the various units can be changed as you go along or they can be changed at the end.

Example: Changing as you multiply.

Multiply 5 times 1 ton, 500 lb, 10 oz

5 x 10 oz = 50 oz which is 3 lb, 2 oz; carry the 3 lb
5 x 500 lb = 2,500 lb + 3 lb = 2,503 lb which is 1 ton, 503 lb; carry 1 ton
5 x 1 ton = 5 tons + 1 ton = 6 tons
6 tons, 503 lb, 2 oz.

EXERCISE: Solve the following items and check your answers in the back of the study unit.

1. Add 1 ton, 600 lb, 15 oz; 6 tons, 35 lb, 4 oz, and 1,900 lb, 12 oz.

2. Multiply 25 lb, 6 oz. by 437

3. Divide 490 lb, 10 oz. by 5

Section III. CONVERSION BETWEEN U.S. AND METRIC UNITS.

Work Unit 4-7. CONVERSION OF LINEAR MEASURE

GIVEN A LINEAR MEASUREMENT IN METRIC UNITS, CONVERT TO U.S. UNITS.

GIVEN A LINEAR MEASUREMENT IN U.S. UNITS, CONVERT TO METRIC UNITS.

Hopefully, the time will come when you will no longer need to change from metrics to the U.S. system or vice versa. But until that time arrives you will need to know how to convert meters to yards, miles to kilometers, inches to centimeters, etc.

The most important conversions to Marines are, involving linear measure, centimeters, meters, kilometers and inches, feet, yards, and miles. Just what is the relationship between a mile and a kilometer, a yard and a meter, or a centimeter and an inch? The relationships are not quite as clear cut as working within the metric system, but with the memorization of a few conversion factors, changing meters to miles or kilometers to yards becomes a simple multiplication or division problem.

Converting metric units to U.S. units. As previously mentioned, the memorization of a few conversion factors wher changing metric units to U.S. units is just a simple division problem. The table below gives various conversions that will be very useful to you. If you memorize them, you will be able to calculate and convert any metric distance to the equivalent U.S. distance. Look at the table and memorize the factors that will be most important to you. Following the table, several examples will be presented illustrating the conversion of the more important units.

* Approximate Conversions From Metric Units

When You Know	Divide by	To Find
centimeters	2.5	inches
centimeters	30	feet
meters	0.9	yards
kilometers	1.6	miles

* All conversion factors are approximations, however; they are accurate enough for use in this course.

Examples: Suppose a target is 1500 meters from you. What would this be in yards? Since a meter is longer than a yard, you know that the answer will be larger than 1500. It is only necessary to remember one factor. If you know that 1 yard = 0.9 meters, the problem can be solved by division.

$$0.9\overline{)1500.00} = 1666.66 \text{ yards}$$

Convert 100 kilometers to miles. From the table you see that 1 mile = 1.6 kilometers, the problem can be solved by division.

$$1.6\overline{)100.0} = 62.5 \text{ miles}$$

A man's height is 185 centimeters. What is his height in feet and inches? From the table you know that 1 foot = 30 centimeters and 1 inch = 2.5 centimeters, the problem can be solved by division.

$$30\overline{)185.} = 6. \text{ feet}$$

$$\underline{180}$$

$$5$$

The remaining 5 centimeters can be divided by 2.5 to obtain inches.

$$2.5\overline{)5.0} = 2. \text{ inches}$$

The man's height is 6 feet 2 inches, which equals 185 centimeters.

Converting U.S. units to metric units. To change U.S. units to metric units is just a simple multiplication problem using the same factors that were used to change metric units to U.S. units. Try to remember the conversion factor and the units they are used with. Several examples will be presented illustrating the conversion of the more important units.

Approximate Conversions From U.S. Units.

When You Know	Multiply by	To Find
inches	2.5	centimeters
feet	30	centimeters
yards	0.9	meters
miles	1.6	kilometers

Examples: Convert 5 miles to kilometers. From the chart you remember that 1 mile = 1.6 kilometers. Then 5 miles must be 1.6 times 5.

$$\begin{array}{r} 5 \\ \times\ 1.6 \\ \hline 30 \\ 5 \\ \hline 8.0 \end{array}$$ kilometers = 5 miles

If a hill on a map is indicated as being 250 feet above sea level, how many centimeters high is this? How many meters? From the chart you know that 1 foot = 30 centimeters. Therefore 250 feet would be 30 times 250.

$$\begin{array}{r} 250 \\ \times\ 30 \\ \hline 7500 \end{array}$$ centimeters

From work unit 4-1, you know that to change centimeters to meters, move the decimal point two places to the left.

7500 centimeters = 75 meters

If you have 45 yards, and you want to know how many meters you have, you simply multiply the 45 yards by .9 to obtain the number of meters.

$$\begin{array}{r} 45 \\ \times\ .9 \\ \hline 40.5 \end{array}$$ meters

To change 17 inches to centimeters, you multiply 17 by 2.5 to get the number of centimeters.

$$\begin{array}{r} 17 \\ \times\ 2.5 \\ \hline 42.5 \end{array}$$ centimeters

In summary, to change from metric units to U.S. units, divide by the conversion factor. Multiply by the conversion factor to change from U.S. units to metric units.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. How many kilometers are in 3 miles?
2. 3000 meters equals how many yards?
3. Change 430 centimeters to inches.
4. Change 430 centimeters to feet.
5. Convert 65 miles to kilometers.
6. 7 1/2 inches equal how many centimeters?
7. 8 feet equal how many centimeters?
8. 500 yards equal how many meters?

Work Unit 4-8. CONVERSION OF CAPACITY MEASURE

GIVEN A CAPACITY MEASUREMENT IN METRIC UNITS, CONVERT TO U. S. UNITS.

GIVEN A CAPACITY MEASUREMENT IN U. S. UNITS, CONVERT TO METRIC UNITS.

Converting metric capacity units to U. S. capacity units. Conversion between metric capacity and U. S. capacity is done the exact same way as shown in work unit 4-7. The only difference will be in the conversion factors. Changing from metric units to U. S. units is just a simple division problem. Look at the table and memorize the conversion factors that apply to the different units. Keep in mind that the conversion factors are only approximations and are not used when a great deal of accuracy is needed. Following the table, several examples will be presented.

Approximate Conversions From Metric Units

When You Know	Divide by	To Find
milliliters	30	fluid ounces
liters	0.47	pints
liters	0.95	quarts
liters	3.8	gallons

Examples: To convert 150 milliliters into fluid ounces, 150 is divided by 30 to obtain 5 fluid ounces.

$$30\overline{)150}\quad 5 \text{ fluid ounces}$$

Convert 35 liters to pints. From the chart you see that 1 pint = 0.47 liters, the problem can be solved by dividing 35 by 0.47

$$0.47\overline{)35.00}\quad 75.46 \text{ pints}$$

Convert 17 liters to quarts, 17 is divided by 0.95. From the chart 1 quart = 0.95 liters

$$0.95\overline{)17.00}\quad 17.89 \text{ quarts}$$

Convert 83.6 liters to gallons. From the chart you see that 1 gallon = 3.8 liters.

$$3.8\overline{)83.6}\quad 22. \text{ gallons}$$

Converting U. S. capacity units to metric capacity units. To change U. S. capacity units to metric units, simple multiplication is used. The approximate conversion factors are the same as the ones used to convert metric capacity to U. S. capacity. The conversion factor should be committed to memory and applied to their respective units. Several examples will be presented following the conversion table.

Approximate Conversions From U.S. Units

When You Know	Multiply by	To Find
fluid ounces	30	milliliters
pints	0.47	liters
quarts	0.95	liters
gallons	3.8	liters

Examples: To convert 50 fluid ounces to milliliters, 50 is multiplied by 30.

50
x 30
1500 milliliters

Change 3 pints to liters. From the table you see that 1 pint = 0.47 liters.

3
x 0.47
1.41 liters

To change 6 quarts to liters, multiply 6 by 0.95 to obtain liters. 1 quart = 0.95 liters.

6
x 0.95
5.70 liters

How many liters are there in a 5 gallon water can? From the table, 1 gallon = 3.8 liters

5
x 3.8
19.0 liters

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. Change 475 milliliters to fluid ounces.
2. Convert 15 liters to pints.
3. 6 liters equal how many quarts?
4. 48 liters equal how many gallons?
5. Convert 64 fluid ounces to milliliters.
6. 4.5 pints equal how many liters?
7. Convert 8.25 quarts to liters.
8. One 55-gallon drum will contain how many liters?

Work Unit 4-9. CONVERSION OF WEIGHT MEASURE

GIVEN A WEIGHT MEASUREMENT IN METRIC UNITS, CONVERT TO U.S. UNITS.

GIVEN A WEIGHT MEASUREMENT IN U.S. UNITS, CONVERT TO METRIC UNITS.

Converting metric weight to U.S. weight. Converting metric weight to U.S. weight is done in the same manner that was covered in the two preceeding work units. The conversion is done with the use of division and some very important conversion factors. The conversion factors in the table are approximations and should not be used for accurate weight conversions. Examples will follow the factor conversion table.

Approximate Conversion From Metric Units

When You Know	Divide by	To Find
grams	28	ounces
kilograms	0.45	pounds
tonne	0.9	short tons (2000 lb)

Examples: To change 16 tonne to tons, divide 16 by 0.9. 1 short ton = 0.9 tonne

$$0.9\overline{)16.00} \quad 17.77 \text{ short tons}$$

Convert 99 kilograms to pounds. From the table you see that 1 pound = 0.45 kilograms.

$$0.45\overline{)99.00.} \quad 220. \text{ pounds}$$

To convert 167 grams to ounces, 167 is divided by 28. 1 ounce = 28 grams.

$$28\overline{)167.00} \quad 5.96 \text{ ounces}$$

Converting U.S. weight to metric weight. To convert U.S. weight to metric weight you multiply the U.S. weight by the conversion factor used to convert metric to U.S. weight. Keep in mind that the conversions are only approximations.

Approximate Conversion From U.S. Units

When You Know	Multiply by	To Find
ounces	28	grams
pounds	0.45	kilograms
short tons (2000 lb)	0.9	tonne

Examples: To change 12 ounces to grams multiply 12 by 28. 1 ounce = 28 grams.

$$\begin{array}{r} 12 \\ \times\ 28 \\ \hline 336 \text{ grams} \end{array}$$

160 pounds can be converted to kilograms by multiplying 160 by 0.45. 1 pound = 0.45 kilograms.

$$\begin{array}{r} 160 \\ \times\ 0.45 \\ \hline 72.00 \text{ kilograms} \end{array}$$

To convert 60 short tons to tonne, multiply 60 by 0.9. 1 short ton = 0.9 tonne.

$$\begin{array}{r} 60 \\ \times\ 0.9 \\ \hline 54.0 \text{ tonne} \end{array}$$

Commit to memory the conversion factors and the units to which they apply. To change from metric units to U.S. units, divide by the conversion factor. Multiply by the conversion factor to convert U.S. units to metric units.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. Convert 182 grams to ounces.

2. 4.5 kilograms equal how many pounds?

3. Convert 18 tonne to short tons.

4. 32 ounces equal how many grams?

5. Change 45 pounds to kilograms.

6. Convert 21 short tons to tonne.

7. 78 pounds equal how many kilograms?

8. 42 grams equal how many ounces?

SUMMARY REVIEW

In the preceding work units you have been introduced to the various units of measurement. You can now convert one metric unit of measurement to another metric unit using the decimal system. When given various units of measure used in the U.S., you can compute these measurements using like columns to line up the units of measure. Using different conversion factors you can convert metric units to U.S. units by simple division. You have used various conversion factors and the multiplication process to convert U.S. units of measure to metric units.

Answers to Study Unit #4 Exercises

Work Unit 4-1.

1. 43,000 centimeters = 430 meters

2. 15 kilometers = 15,000 meters

3. 600 meters = .6 kilometers

4. 7 kilometers = 700,000 centimeters

5. On a map with a scale of 1:50,000, the distance between point A and B is 18cm. What is the actual ground distance in meters?

   1 cm (map) = 50,000 cm (ground)
   18 cm (map) = 50,000 x 18
   900,000 cm on the ground

   To change cm to m, move the decimal point <u>two</u> places to the <u>left</u>.

   900,000 cm = 9,000 meters

6. On a 1:25,000 map, 14.3 centimeters equals how many kilometers?

   1 cm (map) = 25,000 cm (ground)
   14.3 (map) = 25,000 x 14.3
   357,500 cm on the ground
   357,500 cm = 3.575 kilometers

Work Unit 4-2.

1. 155 liters = 155,000 milliliters

2. 755 liters = 7.55 hectoliters

3. 4,834 milliliters = 4.834 liters

4. 6.5 hectoliters = 650 liters
   350 milliliters = .350 liters

   650 liters plus .350 liters = 650.35 liters.

5. 750 milliliters = .750 liters
   4.75 liters minus .750 liters = 4 liters

6. 36 liters = .36 hectoliters
   4,000 milliliters = .04 hectoliters

   .36 hectoliters
   + .04 hectoliters
   .4 hectoliters

Work Unit 4-3.

1. 3,450 kilograms = 3.45 tonne

2. 500 grams = .5 kilograms

3. 430 tonne = 430,000 kilograms

4. 750,000 grams = .75 tonne

5. 60,000 grams = 60 kilograms

   167 kilograms
   -60 kilograms
   107 kilograms

6. 650,000 grams = 650 kilograms
   37 tonne = 37,000 kilograms

   37,000 kilograms
   + 650 kilograms
   37,650 kilograms

Work Unit 4-4.

1. 3 ft 4 in = 3 ft, 4 in.
   27 in = 2 ft, 3 in.
   1 1/2 yd = 4 ft, 6 in.
   + 2 3/4 ft = 2 ft, 9 in.
   11 ft, 22 in. = 12 ft, 10 in.

2. Changing as you multiply

   19 times 5 yd, 2 ft, 27 in.
   19 x 27 in. = 513 in. which is 42 ft, 9 in., carry 42 ft.
   19 x 2 ft = 38 ft + 42 ft = 80 ft which is 26 yd, 2 ft, carry 26 yd
   19 x 5 yd = 95 yd + 26 = 121 yd

   121 yd, 2 ft, 9 in.

3. Divide 23 yd, 1 ft by 10
   23 yd = 69 ft + 1 ft = 70 ft
   70 ft ÷ 10 = 7 ft or 2 yd, 1 ft

Work Unit 4-5.

1. 3 gal, 3 qt, 1 pt
   5 gal, 2 qt
   + 6 gal, 1 qt, 1 pt
   14 gal, 6 qt, 2 pt = 15 gal, 3 qt

2. Multiply 2 gal, 3 qt, by 417
   417 x 3 qt = 1251 qt which is 312 gal, 3 qt, carry 312 gal
   417 x 2 gal = 834 gal + 312 = 1,146 gal

   1,146 gal, 3 qt

3. Divide 6 gal, 3 qt, 1 pt by 11
   6 gal, 3 qt, 1 pt = 55 pt

   55 pts ÷ 11 = 5 pt or 2 qt, 1 pt

Work Unit 4-6

1. 1 ton, 600 lb, 15 oz
   6 ton, 35 lb, 4 oz
   \+ 1,900 lb, 12 oz
   7 ton, 2,535 lb, 31 oz = 8 ton, 536 lb, 15 oz

2. Multiply 25 lb, 6 oz, by 437
   437 x 6 oz = 2,622 oz which is 163 lb, 14 oz, carry 163 lb
   437 x 25 = 10,925 lb + 163 lb = 11,088 lb, which is 5 tons, 1,088 lb

   5 ton, 1,088 lb, 14 oz

3. Divide 490 lb, 10 oz by 5
   490 lb, 10 oz = 7,850 oz
   7,850 oz ÷ 5 = 1,570 oz or 98 lb, 2 oz

Work Unit 4-7.

1. 1 mile = 1.6 kilometers
   3 miles x 1.6 = 4.8 kilometers

2. 1 yard = 0.9 meters
   3,000 meters ÷ 0.9 = 3,333.33 yards

3. 1 inch = 2.5 centimeters
   430 ÷ 2.5 = 172 inches

4. 1 foot = 30 centimeters
   430 ÷ 30 = 14.33 feet

5. 1 mile = 1.6 kilometers
   65 x 1.6 = 104 kilometers

6. 1 inch = 2.5 centimeters
   7 1/2 inches = 7.5 inches
   7.5 x 2.5 = 18.75 centimeters

7. 1 foot = 30 centimeters
   8 feet x 30 = 240 centimeters

8. 1 yard = 0.9 meters
   500 yards x 0.9 = 450 meters

Work Unit 4-8.

1. 1 fluid ounce = 30 milliliters
   475 milliliters ÷ 30 = 15.83 fluid ounces

2. 1 pint = 0.47 liters
   15 liters ÷ 0.47 = 31.91 pints

3. 1 quart = 0.95
   6 liters ÷ 0.95 = 6.31 quarts

4. 1 gallon = 3.8 liters
   48 liters ÷ 3.8 = 12.63 gallons

5. 1 fluid ounce = 30 milliliters
   64 fluid ounces x 30 = 1,920 milliliters

6. 1 pint = 0.47 liters
   4.5 pints x 0.47 = 2.115 liters

7. 1 quart = 0.95 liters
   8.25 quarts x 0.95 = 7.8375 liters

8. 1 gallon = 3.8 liters
   55 gallons x 3.8 = 209 liters

Work Unit 4-9.

1. 1 ounce = 28 grams
   182 grams ÷ 28 = 6.5 ounces

2. 1 pound = 0.45 kilograms
   4.5 kilograms ÷ 0.45 = 10 pounds

3. 1 short ton = 0.9 tonne
   18 tonne ÷ 0.9 = 20 short tons

4. 1 ounce = 28 grams
   32 ounces x 28 = 896 grams

5. 1 pound = 0.45 kilograms
   45 pounds x 0.45 = 20.25 kilograms

6. 1 short ton = 0.9 tonne
   21 short tons x 0.9 = 18.9 tonne

7. 1 pound = 0.45 kilograms
   78 pounds x 0.45 = 35.1 kilograms

8. 1 ounce = 28 grams
   42 grams ÷ 28 = 1.5 ounces

STUDY UNIT 5

GEOMETRIC FORMS

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO MEASURE ANGLES WITH A PROTRACTOR AND COMPUTE A SERIES OF ANGLE READINGS. YOU WILL BE ABLE TO FIND THE AREA AND VOLUME, OF VARIOUS PLANE AND SOLID FIGURES AND FIND THE MISSING SIDE OF RIGHT TRIANGLES BY APPLYING THE LAW OF PYTHAGORAS.

You are patrolling on a compass heading of 78° 50' and arrive at an impassable obstacle. You decide to go around it and return to your original heading by a succession of 90° compass changes. If your first heading is 90° to the right, what will the new heading be?

An area to be used as a supply dump measures 110' by 86'. If it is to be camouflaged with nets measuring 36' by 44', what is the minimum number of nets that should be requisitioned?

What do these seemingly unrelated questions have in common? They all involve some type of measurement or computation with geometric figures. The various geometries are probably the most used form of mathematics (other than arithmetic) in the Marine Corps. Just several examples were presented above. Surely you can think of other geometric applications that apply to your particular job.

Our friend Charlie Caveman probably did not have much use for geometry because the need for it arose with the establishment of cities (architecture) and the owning of property (boundaries). Its use is recorded in the first written records of mankind. The Rhind Papyrus, mentioned previously, has reference to problems involving geometry. Fundamental formulas for measurement were known in early Egypt and Babylonia and it is probably due to the yearly fluctuations of the Nile river that much of geometry was developed. The Egyptian "rope stretchers" using the Pythagorean relationship, which will be discussed later, were the original surveyors who each year determined property lines which had been destroyed by the Nile. Later the Greeks became interested in geometry, and, for many years, it was the primary type of mathematics studied. Applications to navigation and astronomy came from their efforts. Today when geometry is mentioned many people think of the rigorous proofs, of complicated constructions, and of the theorems that had to be memorized. This course will avoid those and stick to the practical applications of some geometric principles.

Section I. ANGLES

Work Unit 5-1. MEASURING ANGLES

GIVEN AN ANGLE, APPLY THE OPERATIONS FOR MEASURING AN ANGLE WITH A PROTRACTOR TO OBTAIN THE DEGREES.

DEFINITION. An angle is a set of points consisting of two rays and a common end-point (fig 5-1). Angle BAC is abbreviated BAC. It can also be written angle CAB; they are the same angle. Point A which is the middle letter in the angle is called the vertex of the angle.

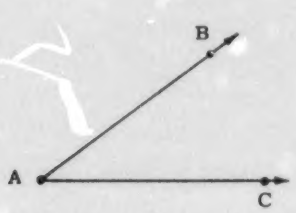


Fig 5-1. Ray AB + ray AC = angle BAC.

An angle is referred to in one of several ways (fig 5-2). It may be called angle B as in (1), angle ABC or angle CBA as in (2), or it may be labeled with a small letter or numeral as in (3) and (4).

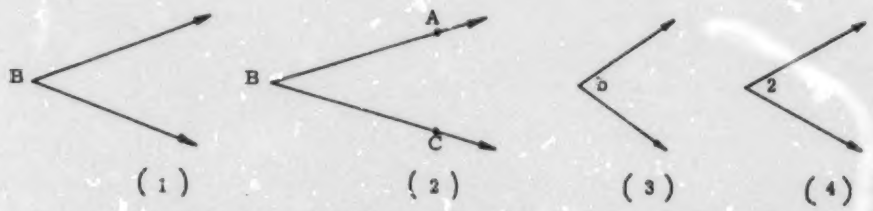


Fig 5-2. Labeling an angle.

Measurement. Although the angle is the set of points on the two rays, the measure of the angle is the amount of "opening" between the rays in the interior of the angle. To measure an angle, we must have a unit of measure and a tool inscribed with those units. The unit is the degree (^{o}) and the tool is the protractor. The degree is determined by a set of 181 rays drawn from the same endpoint. These rays which are numbered from 0 to 180 make 180 angles all the same size. One of these angles is the standard unit or degree for measuring angles. Figure 5-3 illustrates the scale for measuring 180^{o} and, at the lower right, the actual size of 1^{o}. Note that the protractor that comes with the course has two scales. Both start at the left and run to the right. One is graduated from zero to 180 and the other from 180 to zero. This gives each degree on the protractor two values and care must be exercised when reading the measure of the angle from the protractor.

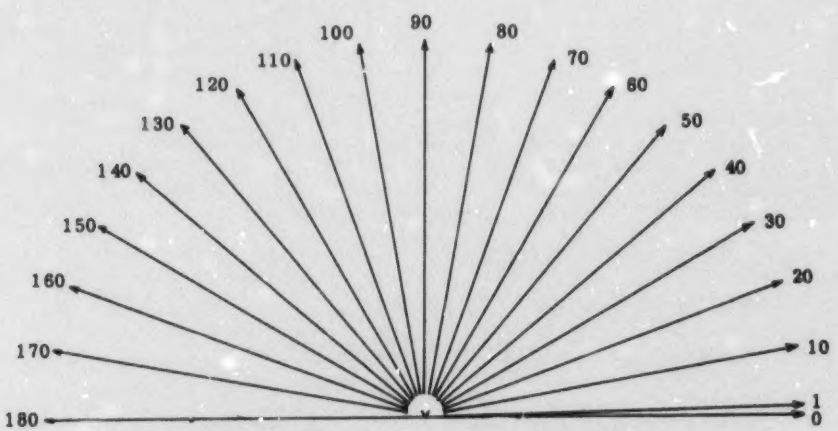
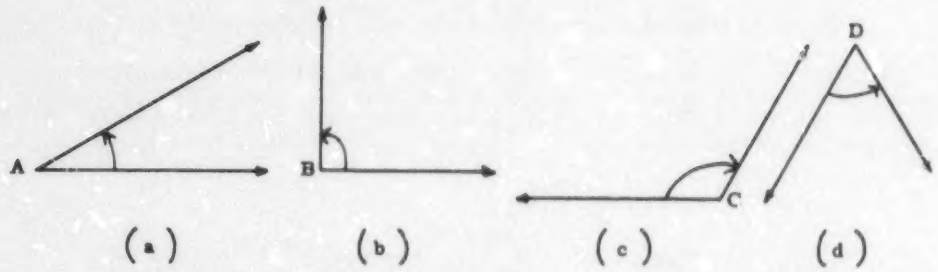


Fig 5-3. Measuring scale for angles.

To measure an angle with the protractor, place it on the angle so that the center point is on the vertex of the angle and the ray which corresponds to zero/180 on the protractor lies on one side of the angle. Then observe the protractor ray which lies on the other side of the angle. The number that corresponds to this ray is the measure, in degrees, of the angle. Remember, each mark on the protractor has two values. Be sure to read from the correct scale when you read the measure of the angle. The amount of "opening" between the rays of the angle, shown by a curved arrow, determines the scale to be used.

Examples:

Use the protractor to measure the angles below. If the parts of the rays are not long enough, lay the edge of a piece of paper along the side of the angle or carefully extend the rays with a pencil.



(a) To measure angle (a), place the protractor so that the center point is on the vertex (A) of the angle and the ray which corresponds to zero/180 on the protractor lies on the bottom ray of the angle. Read up the inside scale (zero to 180) on the right side of the protractor until you read the number that corresponds to the top ray of the angle. The top ray will correspond to 30° on the protractor.

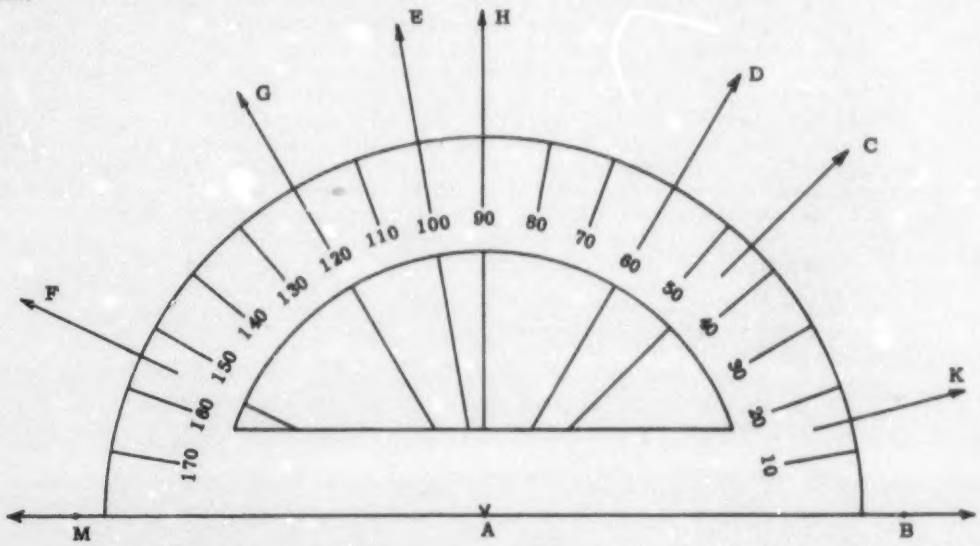
(b) To measure angle (b), place the protractor so that the center point is on the vertex (B) of the angle and the ray which corresponds to zero/180 on the protractor lies on the horizontal ray of the angle. Read up the inside scale (zero to 180) on the right side of the protractor until you read the number that corresponds with the vertical ray of the angle. The vertical ray will read 90° on the protractor.

(c) To measure angle (c), place the protractor so that the center point is on the vertex (C) of the angle and align the left side of the zero/180 line of the protractor on the horizontal ray of the angle. Read up the outside scale (zero to 180) on the left side of the protractor until the other ray corresponds to a number on the protractor. The second ray will read 120° on the protractor.

(d) To measure angle (d), move the angle around and place the center point of the protractor on the vertex (D) of the angle. Align the right side of the protractor's zero to 180 line on the bottom ray. Read up the protractor (direction of arrow) on the inside (zero to 180) scale until the other ray corresponds with a number on the protractor. The reading will be 61° on the protractor.

EXERCISE: In the drawing of the protractor below find the measure of each of the angles indicated using the protractor provided with the course. Check your answers in the back of the study unit.

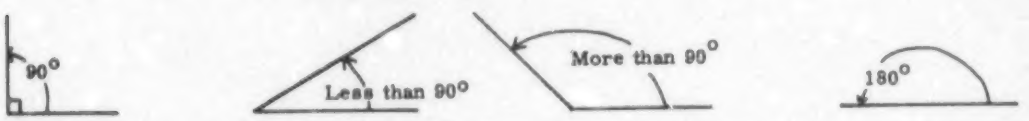
1. BAK
2. MAF
3. KAF
4. BAC
5. GAM
6. HAF
7. BAD
8. MAC
9. BAH
10. CAG



Work Unit 5-2. ANGLE CLASSIFICATIONS

GIVEN AN ANGLE, MEASURE WITH A PROTRACTOR AND CLASSIFY.

Classifications. Angles are classified according to the number of degrees they contain (fig 5-4). An angle of 90° is called a right angle and the rays are said to be perpendicular. An angle less than 90° is called an acute angle. An angle whose measure is more than 90° and less than 180° is called an obtuse angle. An angle of exactly 180° is called a straight angle.

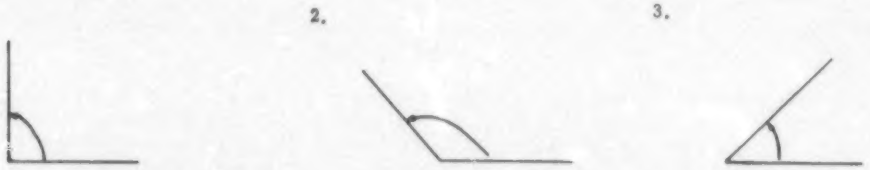


Right angle Acute angle Obtuse angle Straight angle

Fig 5-4. Angle classifications.

EXERCISE: Use the protractor provided with the course to measure and classify the following angles. Check your answers in the back of the study unit.

1. 2. 3.



Work Unit 5-3. COMPUTATION OF ANGLE READINGS

GIVEN TWO OR MORE ANGLE READINGS, COMPUTE INTO DEGREES, MINUTES, AND SECONDS, USING THE OPERATIONS FOR COMPUTATION OF ANGLES.

Computations. So far all of the angles discussed have been even degrees. As you know this is ideal, but usually not the case. Readings on a compass or declination angles on a map may involve parts of a degree. In most instances those parts must be estimated since they are too small to be accurately measured by eye. You must, however, know how to add and subtract the various angles in order to change compass headings, to figure back azimuths, to compute yearly declination, and to accurately plot on a map. To make it more likely that we will reach a destination, all planning of routes is done as accurately as possible. This involves breaking the degree down into minutes (') and seconds ("). Each degree contains 60' and each minute contains 60". You see then that one degree can be broken into 3600" which is far beyond the needs of the average Marine. We will, however, work several problems involving minutes and seconds just for the arithmetic value. Addition and subtraction of angles is done in the same manner as was done with other units of measure and with the decimal system. Keep the proper units in the same column and round off when you have more than 60' or 60".

Examples:

Add $12^{\circ}\ 14'\ 36''$
$22^{\circ}\ 56'\ 42''$
$34^{\circ}\ 70'\ 78''$

You should see that 78" is 1 full minute with 18" remaining. 70' + 1' is 71' which is 1° and 11' remaining.

$34^{\circ} + 1^{\circ}$ is 35° The answer then is $35^{\circ}\ 11'\ 18''$

Add $120^\circ\ 45'\ 22''$
$16^\circ\ 17'\ 38''$
$136^\circ\ 62'\ 60'' = 137^\circ\ 3'$

Subtract $42^\circ\ 12'\ 30''$
$12^\circ\ 30'\ 50''$

You cannot subtract 50 from 30 so as with other units of measure, you must borrow. In this case, 1' or 60" is borrowed and added to 30".

$11'\ 90''$
$42^\circ\ 12'\ 30''$
$12^\circ\ 30'\ 50''$
$40''$

Again you must borrow, this time 1° or 60' and add it to 11',

$41^\circ\ 71'$
$42^\circ\ 11'\ 90''$
$12^\circ\ 30'\ 50''$
$29^\circ\ 41'\ 40''$

Subtract $112^\circ\ 4'\ 25''$
$106^\circ\ 5'\ 24''$

$111^\circ\ 64'$
$112^\circ\ 4'\ 25''$
$106^\circ\ 5'\ 24''$
$5^\circ\ 59'\ 1''$

EXERCISE: Add or subtract as indicated and check your answer in the back of the study unit.

1. Add $18^\circ\ 4'$
180°

2. Add $38^\circ\ 48'$
180°

3. Add $118^\circ\ 29'\ 29''$
$140^\circ\ 42'\ 24''$

4. Add $60^\circ\ 14'\ 2''$
$18^\circ\ 27'\ 17''$

5. Subtract $210^\circ\ 7'$
180°

6. Subtract $274^\circ\ 34'$
180°

7. Subtract $124^\circ\ 17'\ 35''$
$82^\circ\ 18'\ 34''$

8. Subtract $25^\circ\ 16'\ 7''$
$14^\circ\ \ \ 8''$

Section II. PLANE FIGURES

There is an unlimited number of figures in a plane. A plane has no boundaries although we usually visualize it looking like a piece of plywood. For our purposes, we will think of a plane simply as being a flat surface with two dimensions. This piece of paper, if we forget that it actually has a thickness, represents a plane. If you took a pair of scissors and cut out various shapes from this page, each would be a plane figure. Some of these would have configurations that would be usable, others would not have a practical use due to having too many sides that are too irregular in shape. The majority of the objects around you are some regular geometric shape. Let's look at some of the more common figures.

Work Unit 5-4. AREA OF SQUARES

GIVEN THE DIMENSIONS OF A SQUARE, USE THE PROPER FORMULA TO FIND THE AREA

Square. This familiar figure has four sides of equal length and a right angle at each corner. Opposite sides are parallel. (It is assumed that the properties of parallel lines are known. Two lines are parallel if they do not intersect, no matter how far extended.) There are two important measures of a square: the perimeter and the area.

Perimeter. To find the perimeter or distance around a square, add the four sides or even more simply: Perimeter (P) = 4s (length of 1 side). What is the perimeter of a square with a side that measures 16 in.?

$P = 4s$
$P = 4(16)$
$P = 64$ in.

Using equation-solving techniques, if the perimeter is known, the length of a side can be found. For example, if the perimeter of a square is 62 in., what is the length of a side?

$P = 4s$
$62 = 4s$
$\frac{62}{4} = \frac{4s}{4}$

$15\ 1/2 = s$

∴ each side is 15 1/2 in. long.

Area. Area, not only for a square, but all plane figures, is described or measured in terms of square units. A square with sides of 3 in. contains 9 squares, each 1 in. on a side(fig 5-5).

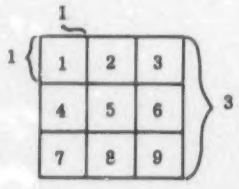


Fig 5-5. Area of a square with s = 3.

You could prove to yourself, by drawing squares of various sizes and marking them off into grids, that the area of a square is equal to the product of two of the sides or more simply the square of one side. $A = s \times s$ or $A = s^2$.

Example: Find the area of a square with a side of 32 in.

$A = s^2$
$A = (32)^2$
$A = 1024$ sq in.

EXERCISE· Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of a square is _____.

2. Find the area of a square with a side of 48 inches.

3. Find the area of a square with a side of 16.5 inches.

Work Unit 5-5. AREA OF RECTANGLES

GIVEN THE DIMENSIONS OF A RECTANGLE, USE THE PROPER FORMULA TO FIND THE AREA.

Rectangle. This is another familiar figure. It has each pair of opposite sides equal in length and parallel, and has a right angle at each corner.

Perimeter. As with the square, the perimeter can be found by adding up the measures of the four sides. Since all are not the same, the longer dimension is called the length (l) and the shorter is called the width (w). There are two lengths and two widths to a rectangle. The perimeter then is: $P = 2l + 2w$.

Example: Find the perimeter of a rectangle 7 in. long and 4 in. wide.

$P = 2l + 2w$
$P = 2(7) + 2(4)$
$P = 14 + 8$
$P = 22$ in.

If the perimeter and one of the two dimensions is known, the missing dimension can be found.

Example: If the perimeter of a rectangle is 82 in. and the length is 31 in., what is the width?

$P = 2l + 2w$
$82 = 2(31) + 2w$
$82 = 62 + 2w$
$20 = 2w$
$10 = w$

∴ The width is 10 in.

If the perimeter of a rectangle is 106 ft and the width is 17 ft, what is the length?

$P = 2l + 2w$
$106 = 2l + 2(17)$
$106 = 2l + 34$
$72 = 2l$
$36 = l$

∴ The length is 36 ft.

Area. Area is described in terms of square units. As with the square, a rectangle can be segmented by a grid and the number of units can be counted (fig 5-6).

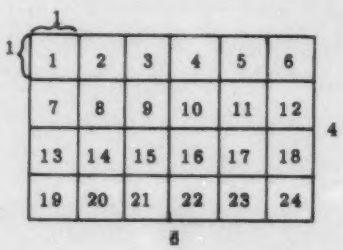


Fig 5-6. Area of a rectangle.

You can see by the figure that the number of square units is equal to the product of the two dimensions of the rectangle. This gives the formula $A = lw$.

Example: Find the area of a rectangle 13.5 in. long and 6.2 in. wide.

$A = lw$
$A = (13.5)(6.2)$
$A = 83.7$

∴ The area is 83.7 sq in.

If you know the area and one of the dimensions, the other dimension can be found.

Example: If the area of a rectangle is 176 sq in. and the length is 16 in., what is the width?

$$A = lw$$
$$176 = 16w$$
$$11 = w$$

$\therefore$ The width is 11 in.

If the area of a rectangle is 417 sq ft and the width is 12 ft, what is the length?

$$A = lw$$
$$417 = l(12)$$
$$417 = 12(l)$$
$$34.75 = l$$

$\therefore$ The length is 34 3/4 ft.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of a rectangle is _____.

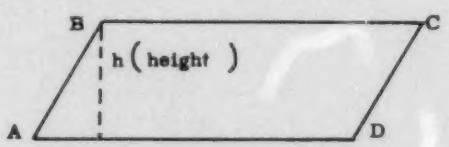
2. Find the area of a rectangle 18 inches long and 12 inches wide.

3. Find the area of a rectangle 22.5 inches long and 10.5 inches wide.

Work Unit 5-6. AREA OF PARALLELOGRAMS

GIVEN THE DIMENSIONS OF A PARALLELOGRAM, USE THE PROPER FORMULA TO FIND THE AREA.

Parallelogram. A parallelogram is a quadrilateral with each pair of opposite sides equal and parallel. It looks like a rectangle that has been shoved sideways (fig 4-10). Opposite angles are also equal. $\angle A = \angle C$ and $\angle B = \angle D$. The long dimension of a parallelogram is usually referred to as the base (b). The perpendicular distance between the bases is called the height (h). Keep in mind that squares and rectangles are in the class of parallelograms, but the form represented in figure 5-7 is the figure usually called parallelogram.



AB = CD AB//CD (// means parallel)
BC = AD BC//AD

Fig 5-7. Parallelogram.

Perimeter. As with the other quadrilaterals, the perimeter is the sum of the lengths of the 4 sides. Two of the sides are called base and let's call the other side the slant width. The formula then would be $P = 2(b) + 2(sw)$.

Area. If we place a grid over a parallelogram as was done with the square and rectangle, due to the sloping sides there would be fractional squares (fig 5-8). By cutting off the three fractional squares on one side and joining them with the parts of squares on the other side, we obtain three complete squares, and a complete rectangle.

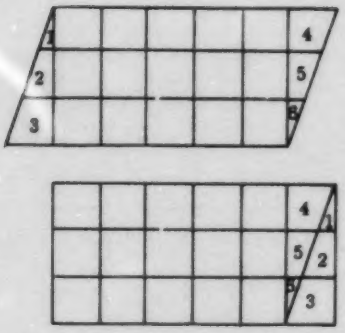
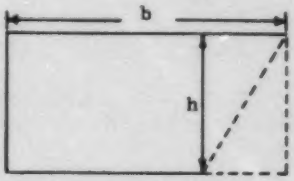


Fig 5-8. Making a parallelogram into a rectangle.

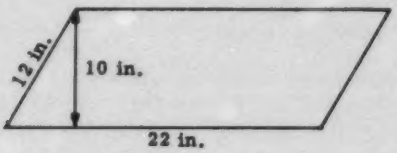
We can borrow the formula for the area of a rectangle and substitute the letters used for parallelograms (fig 5-9).



Area = base x height
A = bh

Fig 5-9. Formula for parallelogram

Example: Find the perimeter and area of the parallelogram illustrated.



$P = 2l + 2w$
$= 2(22) + 2(12)$
$= 44 + 24$
$P = 68$ in.

$A = bh$
$= 22(10)$
$A = 220$ sq in.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of parallelograms is _____.

2. Find the area of a parallelogram with a height of 12 inches and a base of 24 inches.

3. Find the area of a parallelogram with a base of 50.2 inches and a height of 15.3 inches.

Work Unit 5-7. AREA OF TRAPEZOIDS

GIVEN THE DIMENSIONS OF A TRAPEZOID, USE THE PROPER FORMULA TO FIND THE AREA.

Trapezoid. This is a quadrilateral with only one pair of opposite sides parallel (the ~ are not equal). As with the parallelogram, the "top" and "bottom" of a trapezoid are referred to as bases. Since they are of different lengths, the subscripts 1 and 2 are used to differentiate between them. (A subscript is a small number or letter placed to the lower right of the number. Unlike an exponent it has no value, but is simple a means of identification.) In the figure the two bases are read b sub 1 and b sub 2 (fig 5-10). The perpendicular distance between the parallel bases is called the altitude or height.

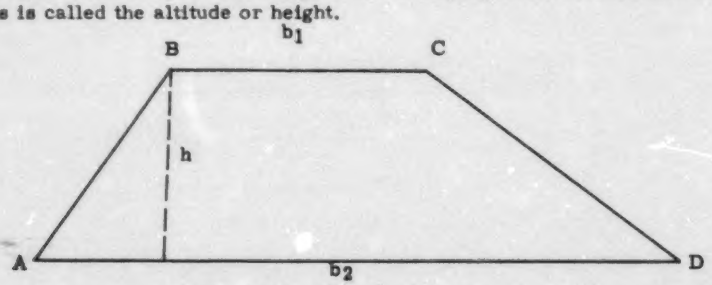


Fig 5-10. Trapezoid.

Perimeter. The perimeter is found by adding the lengths of the four sides. P = a + b + c + d.

Area. The formula for the area of a trapezoid is derived from the formula for the area of a triangle. A trapezoid can be divided into two triangles of unequal area (fig 5-11).

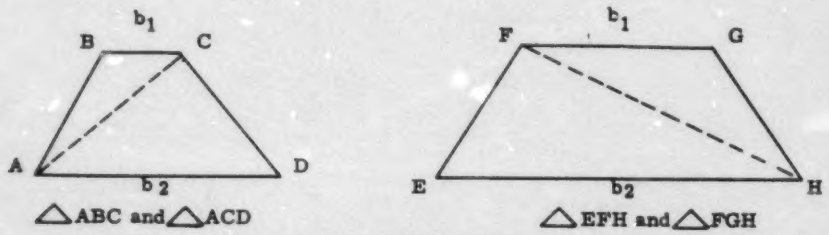


Fig 5-11. Dividing a trapezoid into two unequal triangles.

It should be observed that to find the area of trapezoid ABCD, it is only necessary to add the area of △ABC to the area of △ACD. To find the area of EFGH, add △EFH to △FGH. Rather than have to find two separate areas and then add them, a little algebraic manipulation with the formula for the area of a triangle gives us a formula for the area of a trapezoid. The triangle formula, $A = 1/2\ bh$, will be developed later in the study unit, but will be used here without an explanation. In figure 5-11, the area of ABCD is equal to $1/2b_1h + 1/2b_2h$. Note that although the same height may be used for each triangle each has a different base as indicated by the subscripts. The formula $A = 1/2b_1h + 1/2b_2h$ can be changed to $A = 1/2\ h(b_1 + b_2)$. This is beyond the algebra that was covered in study unit 3, but if you can remember how to factor, you should see how it was derived. Let's look at some examples using the formula.

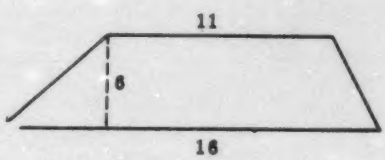
Examples: Find the areas of the trapezoids illustrated.

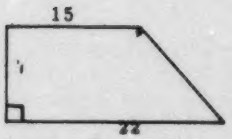
$$A = \frac{1}{2} h \left(b_1 + b_2\right)$$

$$A = \frac{1}{2} (6)\ (11 + 16)$$

$$A = 3(27)$$

$$\underline{A = 81 \text{ sq units}}$$





$A = \frac{1}{2} h (b_1 + b_2)$

$A = \frac{1}{2} (7) (15 + 22)$

$A = \frac{1}{2} (7) (37)$

$A = \frac{1}{2} (259)$

$A = 129 \frac{1}{2}$ sq units

$A = \frac{1}{2} h (b_1 + b_2)$

$A = \frac{1}{2} (8) (19 + 25)$

$A = 4 (44)$

A = 176 sq units

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of a trapezoid is _____.
2. Find the area of a trapezoid, the height is 10 inches, b_1 is 28 inches, and b_2 is 34 inches.
3. Find the area of a trapezoid that has a height of 12 inches with b_1 15.5 inches long, and b_2 23.4 inches long.

Work Unit 5-8. AREA OF TRAIANGLES

GIVEN THE DIMENSIONS OF A TRIANGLE, USE THE PROPER FORMULA TO FIND THE AREA.

Triangles. A triangle ABC is the set of points A, B, and C and all points of AB between A and B, all points of AC between A and C, and all points of BC between B and C. The points A, B, and C are called the vertices (plural of vertex) of the triangle. A triangle may also be thought of as points A, B, and C, not on the same straight line, and the line segments that connect them. Note that in both of these descriptions no mention is made of the inside or outside areas. The triangle is only that part represented by the line. All inside is the interior of the triangle, all outside is the exterior (fig 5-12).

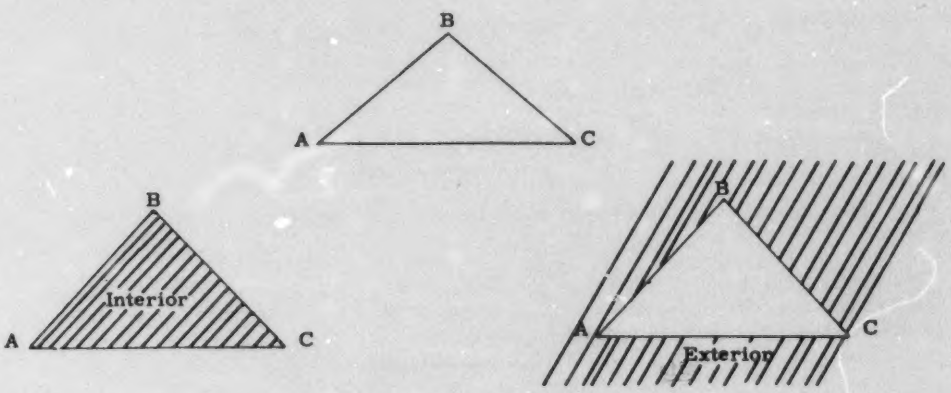
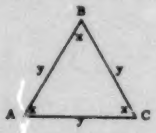


Fig 5-12. A triangle, its interior, and its exterior.

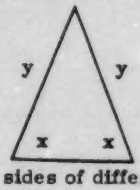
Types. A triangle can be classified according to the lengths of its sides or the measures of its angles. Some of the types are:

Equilateral - As the name implies, equal sides. All three sides are the same length.



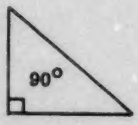
Equiangular - Equal angles. This and equilateral are the same. If all of the sides are the same length, then all of the angles are the same measure (60°). The sum of the measures of the angles of a triangle is always 180°.

Isosceles - A triangle with two equal sides, and consequently two equal angles. The equal angles are opposite the equal sides.

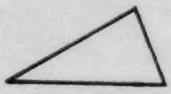


Scalene - A triangle having all three sides of different length. All three angles are also of different measure.

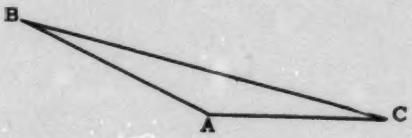
Right triangle - A triangle having one of its angles measuring 90°.



Acute - All three angles have measures less than a right angle.

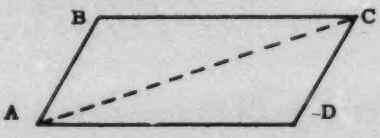


Obtuse - One of the three angles has a measure greater than a right angle. $\angle A$

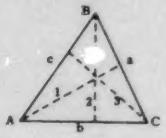


Perimeter. The perimeter of a triangle is found by adding the lengths of the three sides. $P = a + b + c$.

Area. The formula for the area of a triangle can be derived from the area of a parallelogram. Any two equal triangles form a parallelogram. In the illustration, note that the parallelogram has been divided into two equal triangles ABC and ACD.

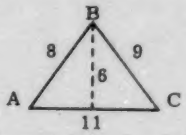


Recall that the formula for the area of a parallelogram is A = bh. Since a parallelogram contains two equal triangles, the area of one of them would be half that of the parallelogram or 1/2 bh. It matters not which dimension you call the base or which the height. For example in the illustration below if b is used as the base, 2 is the height; if c is the base, 3 is the height; and if a is the base, then 1 is the height. With real dimensions, any of the combinations would give the area of the triangle.



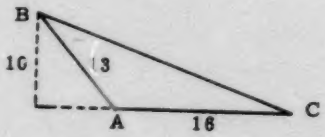
Example: Find the area of the triangle illustrated.

A = 1/2 bh
A = 1/2 x 11 x 6
A = 33 sq units



Find the area of the triangle illustrated.

A = 1/2 bh
A = 1/2 x 16 x 10
A = 80 sq units



EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of a triangle is _____.

2. Find the area of a triangle with a base of 20 inches and a height of 15 inches.

3. A triangle with a height of 16.5 inches and a base of 22.4 inches will have a total area of _____ square inches.

Work Unit 5-9. CIRCUMFERENCE OF CIRCLES

GIVEN THE DIAMETER OF A CIRCLE, USE THE PROPER FORMULA TO FIND THE CIRCUMFERENCE.

Circles. A circle is a closed curve all of those points are the same distance from a fixed point called the center. The center is usually labeled with a capital letter and the circle can be referred to by this point (fig 5-13). As with the other plane figures, a circle is only the points on the line that represents the circle. All outside is the exterior; all inside the closed curve is the interior. The set of points that is the circle is called the circumference. This is to the circle as the perimeter is to quadrilaterals. The fixed distance from the center to any point on the circle is called the radius. A line segment having its endpoints on the circle is called a chord. It does not necessarily pass through the center. If it does, it is called the diameter of the circle. A diameter is equal to the length of two radii (d = 2r). An important

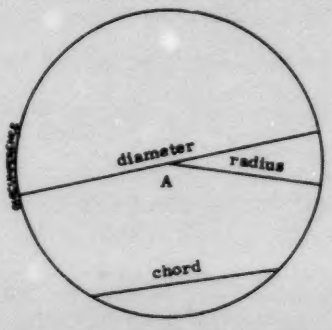


Fig 5-13. Nomenclature of a circle.

concept about circles is that the ratio of the circumference of a circle to its diameter is a constant value 3.141592. It is symbolized by the Greek letter π (pi), and has been known and used since ancient times. You could derive the value for yourself with a simple experiment. Take a C-ration can, a round of ammo, a cup, a roll of tape, or any other circular object. Measure the circumference and the diameter of at least three of them and average the quotients of the ratio c/d. You may not get exactly 3.14 due to errors in measurement and the measuring device, but it should be very close. Pi is usually rounded off to the decimal 3.14 or the fractions 3 1/7 or 22/7.

Circumference. Since circumference has been defined and its relationship to π and the diameter has been shown, it should be easy to derive the formula for determining the circumference of any circle.

If $c/d = \pi$, then through manipulation of this equation we can find the value of c.

$$\frac{1}{d} \cdot \frac{c}{1} = \pi \, d$$

$$c = \pi \, d$$

Since $d = 2r$, the formula could also be $c = 2\pi r$.

Example: Find the circumference of a circle with a radius of 5 in.

$C = 2\pi r$
$C = 2 (3.14) 5$
$C = 31.4$ in.

Find the circumference of a circle with a diameter of 14 in.

$C = \pi d$
$C = 3.14 (14)$
$C = 43.96$ in.

Note this problem could have been done using the fractional π.

$C = 2\pi r$
$C = 22/7 \times 14$
$C = 44$ There is a difference of .04 in the answers, but this is permissible.

Find the radius of a circle that has a circumference of 56.52 in.

$C = 2\pi r$
$56.52 = 2 (3.14) r$
$56.52 = 6.28 r$
$9 = r$ The radius is 9 in.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the circumference of a circle is $C = 2\pi r$ or _____.

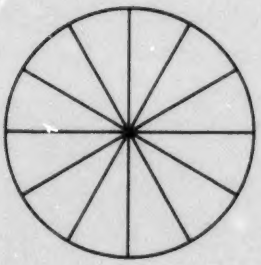
2. Find the circumference of a circle with a radius of 7.5 inches.

3. Find the circumference of a circle with a diameter of 19 inches.

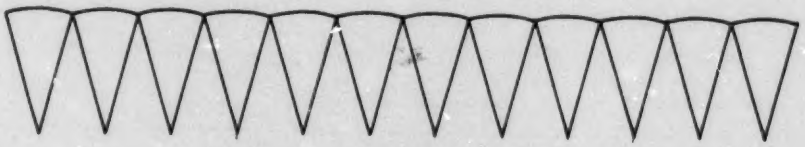
Work Unit 5-10. AREA OF CIRCLES

GIVEN THE RADIUS OF A CIRCLE, USE THE PROPER FORMULA TO FIND THE AREA.

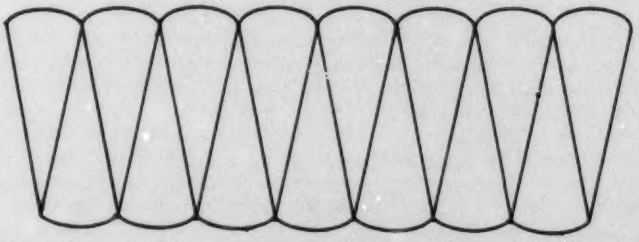
Area. This as always is measured in square units. It would be extremely difficult to determine the number of whole units by placing a grid over the circle as with rectangles, so, as you know, a formula is used. The formula was derived from the rectangle area formula, and can best be described through an illustration (fig 5-14).



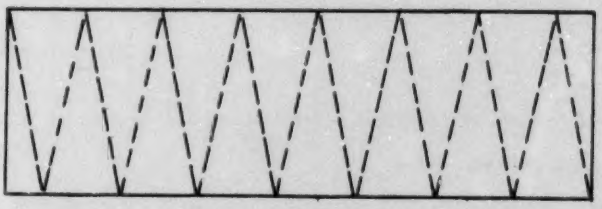
Divide a circle into any number of (preferably large) equal-size pie shaped sections.



Open the circle so that the sections are in a line.



Interlock the sections so that a figure approaching a rectangle is formed.



If the circle could be separated into enough sections, the smaller they got, the flatter the arcs would be. Eventually the figure would be a rectangle.

Fig 5-14. Deriving the area of a circle

If the appropriate parts of the circle are substituted into the formula for the area of a rectangle, we will get a circle area formula. The length of the rectangle that has been formed is related to the circumference of the circle. Remember that the top and bottom of the rectangle were the arcs that were originally the circumference. Since the circumference is $2\pi r$, the length of a rectangle would be half of this or

$1/2 \times 2\pi r$. The width of the rectangle would be the radius of the circle.

$A = lw$

$A = 1/2 \cdot 2\pi r \cdot r$

$A = \pi r \cdot r$

$A = \pi r^2$ This is the formula for the area of a circle.

Examples: Find the area of a circle with a radius of 5 in.

$A = \pi r^2$
$A = 3.14 (5^2)$
$A = 3.14 (25)$
$A = 78.5$ sq in.

Find the area of a circle with a diameter of 14 in.

Note that diameter is given instead of radius.

$A = \pi r^2$
$A = 3.14 (7^2)$
$A = 3.14 (49)$
$A = 153.86$ sq in.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the area of a circle is _____.

2. Find the area of a circle with a radius of 3.5 inches.

3. Find the area of a circle with a diameter of 22 inches.

Section III. SOLID FIGURES

Probably a more appropriate title for this next group of geometric figures would be "three dimensional." They are called solid figures because we are concerned with the points that make up the complete figure. Even though it may be a hollow box, there are points in this "hollow" area that help to form the solid figure. The unit of measure for solid figures is a cube one unit of length on each edge (fig 5-15). When finding the volume of a solid figure, you are finding out how many 1-unit cubes or cubic units are contained in the figure. Notice that a third dimension has been added. Whereas in plane figures we dealt with base and height or length and width, we now have length, width, and height contained in one figure.

Fig 5-15. Standard of measure for solid figures. A one-unit cube.

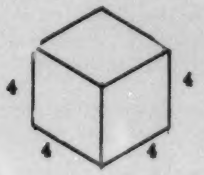
Work Unit 5-11. VOLUME OF CUBES

GIVEN THE DIMENSIONS OF A CUBE, USE THE PROPER FORMULA TO FIND THE VOLUME.

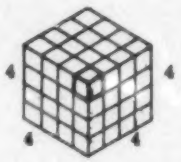
Cube. The familiar cube as illustrated below is a geometric figure with six square faces. The edges all have the same measure. The sum of the areas of the six faces is called the surface area.

Example: A cube is 4 in. on each edge. What is the surface area of the cube? There are 6 faces, each 4 x 4 in. area.

Surface area = $6 (4^2)$
= 6 (16)
Surface area = 96 sq in.

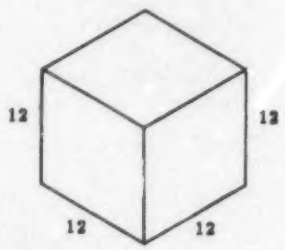


To find the volume of a cube we must find the number of unit cubes contained in the figure.



Example: Find the volume of a cube 4 in. on edge. By drawing several cubes or by taking one 1-cubic-inch block and constructing larger cubes, it would not take long to generalize the formula for the volume of a cube: V = lwh. Remember, though, that a cube has the unique characteristic of all square faces, hence l, w, and h (the edges) are the same. Consequently the formula can be changed to V = e·e·e or V = e^3. The volume of the cube then is:

$V = e^3$
$V = 4^3$
V = 64 cu in.



Find the number of cubic inches in a cubic foot. First you must change each 1-foot edge to 12 in., then simply plug it into the formula.

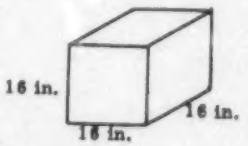
$V = e^3$
$V = 12^3$
V = 1728 cu in.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the volume of a cube is _____.

2. Find the volume in cubic inches of a cube 7 inches on edge.

3. Compute the volume in cubic inches of the cube illustrated.



Work Unit 5-12. VOLUME OF RECTANGULAR SOLIDS

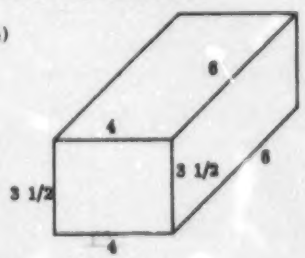
GIVEN THE DIMENSIONS OF A RECTANGULAR SOLID, USE THE PROPER FORMULA TO FIND THE VOLUME.

Rectangular solid. This is similar to the cube, except that the faces are not necessarily squares. Surface area is found by using the same formula. Volume is found by using the original formula that was presented in discussing the cube: V = lwh.

Examples: Find the surface area and volume of a box 6 ft long, 4 ft wide, and 3 1/2 ft high.

The surface would be made up of the two ends of the box, each 3 1/2 x 4; the two sides of the box, each 6 x 3 1/2; and the top and bottom of the box, each 6 x 4.

Surface area = 2(3 1/2 x 4) + 2 (6 x 3 1/2) + 2 (6 x 4)
= 2 (14) + 2 (21) + 2 (24)
= 28 + 42 + 48
Surface area = 118 sq ft



V = lwh
V = 6·4·3 1/2
V = 84 cu ft

How many cu in. would be contained in this box?

Using the figure from the example with the cube there are 1728 cu in. in one cubic foot. Therefore in this box there would be:

84 x 1728 = 145, 152 cu in.

Example: A box has a height of 30 in., a width of 27 in. and a length of 48 in. What is the volume?

$V = lwh$
$V = 48 \cdot 27 \cdot 30$
$V = 38,880$ cu in.

What is the volume in cu ft? One cubic foot is equivalent to 1728 cu in. We have a box with 38,880 cu in. To find the number of cu ft, divide:

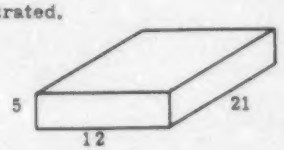
$$\frac{38,880}{1728} = 22.5$$

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the volume of a rectangular solid is ___ _____.

2. Find the volume in cubic feet of a box with a length of 34 in., a width of 17 in., and a height of 16 inches.

3. Compute the volume, in cubic inches, of the figure illustrated.



Work Unit 5-13. VOLUME OF CYLINDERS

GIVEN THE DIMENSIONS OF A CYLINDER, USE THE PROPER FORMULA TO FIND THE VOLUME.

Cylinder. This is the geometric figure typified by a tin can. There are two kinds of cylinders, right cylinders and oblique cylinders (fig 5-16). Since we rarely see oblique cylinders, the discussion will be limited to right cylinders.

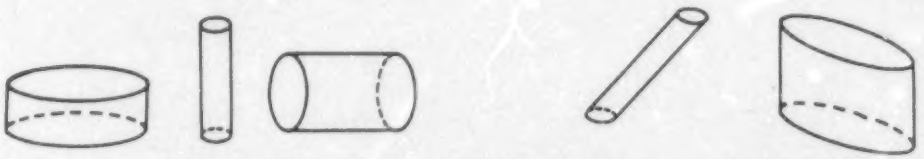


Fig 5-16. Cylinders

A right cylinder has two equal circular bases and a side or lateral surface perpendicular to the bases. We are interested in three facts about cylinders: lateral surface area, total surface area, and volume. All three are based on principles that have been previously discussed. Lateral surface area does not include the top and bottom of the cylinder. If the cylinder is cut vertically and opened up it will form a rectangle with h for one dimension and $2\pi r$ (the circumference of the top or bottom) for the other. Lateral surface, then, would be equal to $2\pi rh$.

Example: Find the lateral surface area of a cylinder with a radius of 7 in. and a height of 14 in.

Lateral surface = 2 x 3.14 x 7 x 14
= 615.44 sq in.

Total surface area would be equal to the lateral surface plus the area of both bases (top and bottom).

$2\pi rh + 2(\pi r^2)$

Through some algebraic manipulation this can be simplified to:

Surface area = $2\pi r(r + h)$

Example: Find the surface area of the cylinder illustrated above.

Surface area = 2 x 3.14 x 7(7 + 14)
= 43.96 (21)
Surface area = 923.16 sq in.

The procedure for finding the volume of a cylinder is comparable to the rectangular solid. Remember that when using lwh that you are finding the area (lw) of the base of the rectangle and then multiplying it by the number of units of height (h). To apply this to cylinders, the area of the base is πr^2. This time the height is $\pi r^2 h$ which is the volume of a cylinder.

Example: Find the volume of the cylinder illustrated above.

$V = \pi r^2 h$
$V = 3.14 \times 7^2 \times 14$
$V = 3.14 \times 49 \times 14$
$V = 2154.04$ cu in.

EXERCISE: Work the following items and check your answers in the back of the study unit.

1. The formula used to find the volume of a cylinder is _____.

2. Find the volume in cubic inches of a cylinder that has a radius of 9.5 inches and a height of 40 inches.

3. Find the volume in cubic inches of the figure illustrated.

Section IV. SIDES OF TRIANGLES

Back in study unit 3, in the discussion of proportion, Pvt Buttplate discovered a method for finding the height of a tree by measuring its shadow and relating it to the height and the length of the shadow of some know object. Buttplate used a proportion to solve his problems, and unknowlingly he was applying a principle that involves the relationship between sides and angles of similar triangles.

There are several relationships between triangles, particularly right triangles, that do not involve mathematical rigor which can be of use to you in many situations. One is the above-mentioned relationship of similar triangles, the other is the Pythagorean property. The Pythagorean law or property is a basic tool in geometric and trigonometric calculations. It has been known since approximately 500 BC when the Greek philosopher-mathematician Pythagoras proved the relationship of the sides of a right triangle. The scuttlebutt is that one day Pythagoras was staring at a mosaic floor. The patterns so intrigued him that he started breaking them up in his mind's eye into right triangles. He noticed that when a square was constructed on each side of a triangle that the areas of the squares had a particular relationship (fig 5-17).

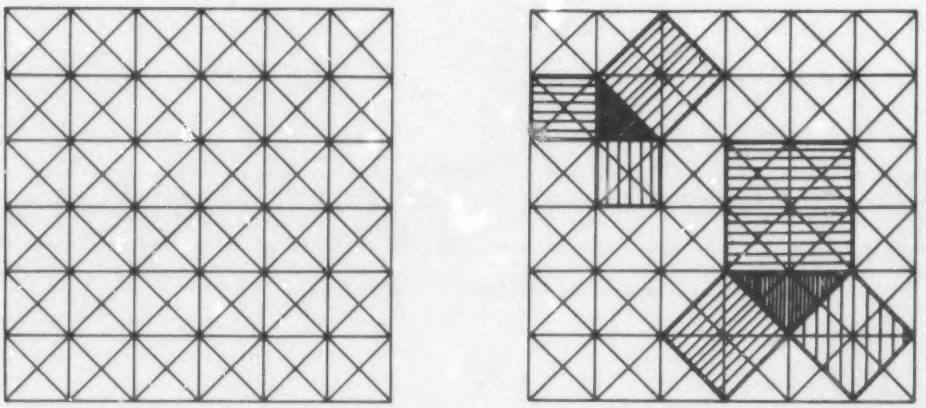


Fig 5-17. The visions of Pythagoras.

What Pythagoras noticed was that if he added the squares of the two shorter sides of a right triangle, the sum was equal to the square of the long side (hypotenuse) of the triangle. This is symbolized in an equation as $a^2 + b^2 = c^2$ where a, b, and c are the lengths of the sides of a right triangle and c is the hypotenuse.

This relationship was mentioned at the beginning of this study unit when reference was made to the rope stretchers. These Egyptian surveyors used the Pythagorean theorem applied to a piece of rope to establish right angles or corners or property. Their rope had knots tied at even intervals. Using the relationship 3:4:5, they could form a right triangle and a perfect 90° angle (fig 5-18).

Fig 5-18. Knots in a rope used to form a 90° angle.

In order to solve problems using the Pythagorean property, you must know how to find the square root of a number.

Work Unit 5-14. SQUARE ROOT

GIVEN A NUMBER, USE THE PROPER OPERATIONS TO FIND THE SQUARE ROOT.

Square root. The square root of any number, x, is that number which multiplied by itself gives x. It can be found in several ways. Some square roots are used so often that they become a matter of common knowledge just as the addition and multiplication tables. For example 144 is the square of 12. Twelve then is the square root of 144 (12 x 12 = 144). The squares of integers from 1 to 20 are used enough to be memorized. If you know the squares, then consequently you know the square root of these squares (fig 5-19).

n	n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

Fig 5-19. Squares of n from 1-20.

There are tables in many textbooks that give the square root of every number up to 1,000, some go even higher. When tables are not available, the computational method is used. There is no "new" method for computing square root, just a fairly rigorous method that has been used for years that needs practice to be maintained.

Example: Find the square root of 1156.

(1) The square root symbol over the number is called a radical sign. $\sqrt{1156}$

(2) Starting at the decimal point, group the digits in pairs. $\sqrt{11\,56.}$

(3) There will be one digit in the answer for every group under the radical. Over the first group, put the largest number whose square does not exceed 11. Three squared is 9, 4^2 is 16. The largest that can be used is 3. Square 3 and write it under the first group.

$$\begin{array}{r} 3 \\ \sqrt{1156} \\ \underline{9} \end{array}$$

(4) Subtract the 9 from 11 and bring down the next group.

$$\begin{array}{r} 3 \\ \sqrt{1156} \\ \underline{9} \\ 256 \end{array}$$

(5) Double the part of the root already found (3 doubled is 6) and add a zero to form a trial divisor. Place this next to 256.

$$\begin{array}{r} 3 \\ \sqrt{1156} \\ \underline{9} \\ 60\ 256 \end{array}$$

(6) Divide 256 by the trial divisor and write the quotient above the second group as the probable second figure of the root.

$$\begin{array}{r} 3\ 4 \\ \sqrt{1156} \\ \underline{9} \\ 60\ 256 \end{array}$$

(7) Correct the trial divisor by adding to it the second number of the root. Then multiply the corrected divisor by the second number of the root and subtract the product, writing any remainder below. In this case, it comes out even. The square root of 1156 is 34. This can be checked by squaring 34.

```
   3 4
√1156
    9
60 256
64 256
```

Find the square root of 2.

(1) There is one group consisting of one digit. Since we know that 2 is not a perfect square, let us carry this to three decimal places. This means that we must add three groups of zeros. The decimal point is placed directly over its original place.

```
  .
√2.000000
```

(2) The highest number whose square does not exceed 2 is 1. The square of 1 is 1.

```
  1.
√2.000000
  1
  1 00
```

(3) Subtract 1 from 2 and bring down the next group.

(4) Double the part of the root already found and add a zero to give a trial divisor of 20.

```
  1.
√2.000000
  1
20 1 00
```

(5) Divide 100 by the trial divisor 20. It goes evenly 5 times, but remember that the divisor must be corrected so 5 is too much. The correct number is 4. Place this as the second figure in the root and add it to the trial divisor.

```
  1. 4
√2.000000
  1
~~20~~ 1 00
24
```

(6) Multiply the correct divisor by 4 and subtract the product from 100. Bring down the next group.

```
  1. 4
√2.000000
  1
~~20~~ 1 00
24   96
     400
```

(7) Double the part of the root found so far and add a zero. 280

(8) Divide 400 by the trial divisor. It will go one time. Correct the trial divisor by adding one and place one as the third figure in the root. Multiply the corrected divisor by one.

```
  1. 4 1
√2.000000
  1
~~20~~ 1 00
24   96
~~280~~ 400
281  281
```

(9) Subtract and bring down the next group. Double the part of the root found so far and add a zero.

```
  1. 4 1
√2.000000
  1
~~20~~ 1 00
24   96
~~280~~ 400
281  281
2820 11900
```

(10) Divide by the trial divisor 2820. Try 4. Place 4 as the fourth figure of the root. Correct the trial divisor by adding 4. Multiply the divisor by 4.

```
      1. 4 1 4
   √2. 000000
~~20~~  1
 24  1 00
       96
 ~~280~~  400
 281   281
 2820  11900
 ~~2824~~ 11296
```

(11) There is a remainder, but we only wish to find the root to three places.

We say then that the square root of 2 is 1.414. It could be carried further. Keep in mind that if you square 1.414, you will not get 2. It will be close and the more decimal places that the root is carried, the closer to two you would get. Three places is close enough.

Find the square root of 912.04.

(1) Group the digits starting at the decimal point. Note that the left-most group has only one digit.

$\sqrt{912.04}$

(2) The highest number whose square does not exceed 9 is 3. Square the 3 and subtract it from 9. Bring down the next group.

```
    3
√ 912.04
    9
   12
```

(3) Double the part of the root found and add zero. 60. This trial devisor is too large for 12.

```
     3
√ ~~9~~ 12.04
     9
 60  12
```

(4) Place a zero as the second figure of the root and bring down the next group. Double the part of the root found so far and add a zero. This is an important step that is often overlooked.

```
     30.
√ ~~9~~ 12.04
     9
 ~~60~~  1204
 600
```

(5) Divide 1204 by the trial divisor 600. It will go twice. Add two to the trial divisor, place 2 as the third figure in the root, and multiply the corrected divisor by 2.

```
     3 0. 2
√ ~~912.04~~
     9
 ~~60~~  12 04
 ~~600~~ 12 04
 602
```

Since it comes out even, the answer is 30.2. It is hoped that these three detailed examples have covered enough situations to enable you to compute the square root of any number. From here on, it is a matter of practice.

Exercise: Compute the following square roots and check your answers in the back of the study unit. If you have trouble, check back to the examples and then try again. Carry answers to two decimal places.

1. $\sqrt{1038}$

2. $\sqrt{511225}$

3. $\sqrt{33}$

4. $\sqrt{94.8676}$

Work Unit 5-15. MISSING SIDE OF TRIANGLE

GIVEN THE DIMENSIONS OF TWO SIDES OF A TRIANGLE, USE THE PROPER FORMULA TO FIND THE DIMENSION OF THE UNKNOWN SIDE.

Application to right triangles. Now that we have discussed the principles of the Pythagorean property and the computation of square root, let us put them together in the solution of problems involving the sides of right triangles. Just for review, the property stated in words is: The sum of the squares of two sides of a right triangle equals the square of the hypotenuse.

Example: Two sides of a right triangle are 9 in. and 12 in. Find the length of the hypotenuse.

Substitute the values in the formula and solve the equation.

$a^2 + b^2 = c^2$

$9^2 + 12^2 = c^2$

$81 + 144 = c^2$

$\sqrt{225} = c$

$15 = c$

Here since $c^2 = 225$, to find c we must take the square root of c^2. If we find the square root of the right member, then we must also find the square root of the left member.

The hypotenuse is 15 in.

What is the distance from home plate to 2d base on a softball field? The bases are 60 ft apart.

Note from the illustration that c, the unknown distance, is the hypotenuse of a right triangle, the right angle being at 1st base.

$a^2 + b^2 = c^2$

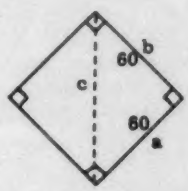
$60^2 + 60^2 = c^2$

$3600 + 3600 = c^2$

$7200 = c^2$

$\sqrt{7200} = c$

$84.85 = c$



The distance is 84.85 ft.

If the hypotenuse and one of the sides is known, the missing side can be found.

Example: The hypotenuse of a right triangle is 13 ft and one side is 5 ft. Find the third side.

$a^2 + b^2 = c^2$

$a^2 + 5^2 = 13^2$

$a^2 + 25 = 169$

$a^2 = 144$

$a = \sqrt{144}$

$a = 12$

The third side is 12 ft.

Exercise: Find the missing sides of the right triangles listed. Check your answer in the back of the study unit. Carry answers to two decimal places.

1. a=5 b=6
2. a=3 b=9
3. c=26 a=24
4. c=39 b=15

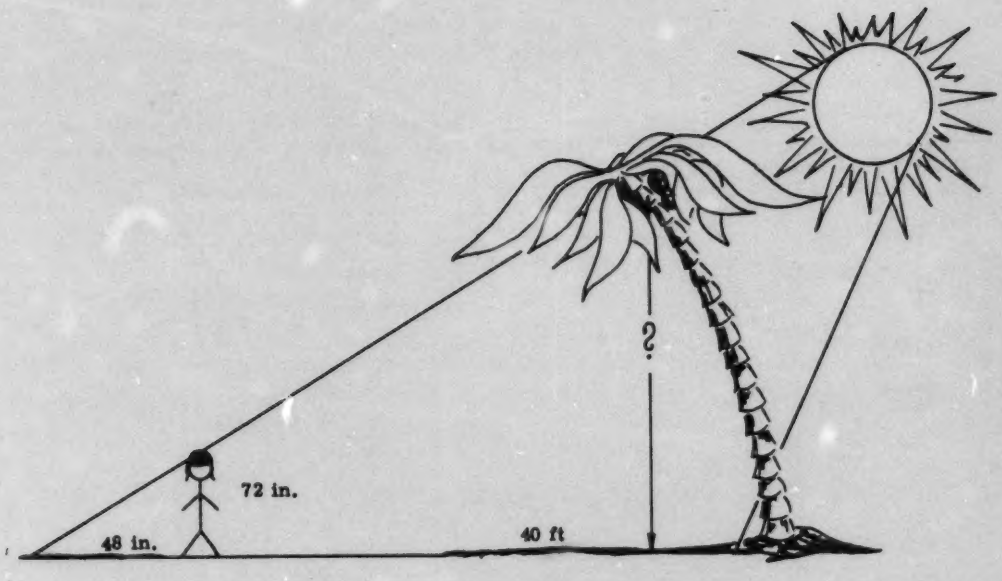
Work Unit 5-16. SIMILAR TRIANGLES

GIVEN THE DIMENSIONS OF TWO SIDES OF ONE TRIANGLE AND THE DIMENSIONS OF ONE SIDE OF ANOTHER TRIANGLE, USE THE PROPER OPERATIONS TO FIND THE DIMENSIONS OF THE MISSING CORRESPONDING SIDE.

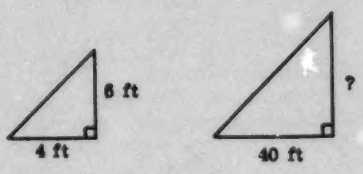
If the corresponding angles of two triangles are equal, the triangles are said to be similar. The triangles do not have to be the same size, that is, have sides the same length; only angles need be the same (fig 5-20).

Fig 5-20. Similar triangles.

In similar triangles, corresponding sides are proportional. Therefore if two sides of one triangle and one side of another triangle are known, the length of the missing corresponding side can be found. This can be illustrated by referring to the problem about the height of the tree in study unit 3 (fig 5-21). The tree and any of the individuals being measured are assumed to be at right angles to the ground. The sun hits both the tree and a person at the same angle. Since the angles are the same measure, the sides are proportional. The lengths of the shadows are known, corresponding to sides of a triangle. The length of another side of one triangle is known (the height of a person). Therefore a proportion is set up and the missing dimension is completed. This, of course, was correctly computed previously. Here, it is just being shown that the reason the answer was correct is because of the theorem that if corresponding angles of two triangles are equal, the triangles are similar and therefore the corresponding sides are proportional.



$$\frac{4}{40} = \frac{6}{x}$$
$$4x = 240$$
$$x = 60$$

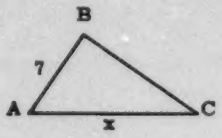


The tree is 60 ft tall.

Fig 5-21. Illustration of problem involving similar triangles.

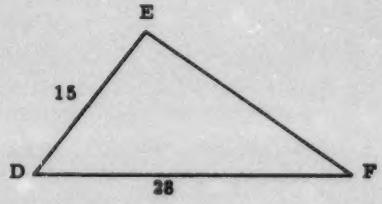
Example: Find side x if $\triangle ABC$ is similar to $\triangle DEF$.

$$\frac{7}{x} = \frac{15}{28}$$
$$15x = 196$$
$$x = 13\frac{1}{15}$$

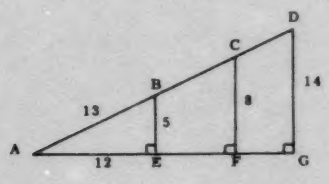


or

$$\frac{7}{15} = \frac{x}{28}$$
$$15x = 196$$
$$x = 13\frac{1}{15}$$



Example: Find the missing sides in △ ACF and △ ADG.



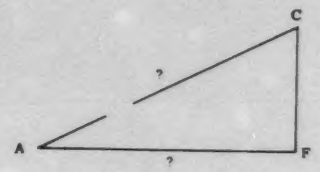
Since ∠A is part of each △ and each has a right angle, the third angles also must be equal. Therefore the triangles are similar. The three sides of △ ABE are given and one side of △ ACF and △ ADG, so proportions can be set up to find the missing sides.

Examples:

$\frac{13}{5} = \frac{\overline{AC}}{8}$

$5\ \overline{AC} = 104$

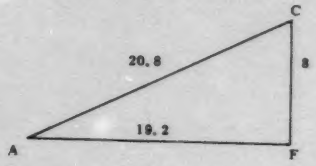
$\overline{AC} = 20.8$



$\frac{12}{5} = \frac{\overline{AF}}{8}$

$5\ \overline{AF} = 96$

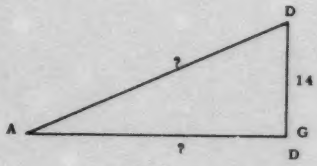
$\overline{AF} = 19.2$



$\frac{13}{5} = \frac{\overline{AD}}{14}$

$\overline{5AD} = 182$

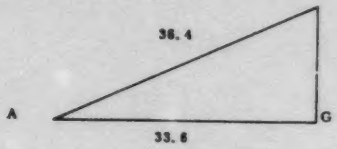
$\overline{AD} = 36.4$



$\frac{12}{5} = \frac{\overline{AG}}{14}$

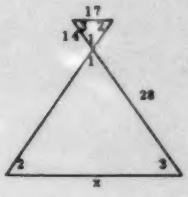
$5\ \overline{AG} = 168$

$\overline{AG} = 33.6$

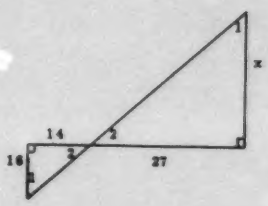


Exercise: Find the missing sides of the triangles. Check your answer in the back of the study unit.

1. Find x.



2. Find x.



SUMMARY REVIEW

In the preceding work units you have been introduced to angles, and you have applied the operations for measuring an angle with a protractor to obtain the degrees. You can now classify an angle, and compute angle readings into degrees, minutes, and seconds. You can use the proper formula to find the area of various plane figures. When given the dimensions of various solid figures, you can use the correct formula to find the volume. You can find the area of a circle when only the radius is known and compute the circumference with a formula when the diameter of a circle is given. You can apply the operations to find the square root of any given number and find the missing side of right triangles using the law of Phythagoras.

Answers to Study Unit # 5 Exercises

Work Unit 5-1.

1. 15°
2. 25°
3. 140°
4. 45°
5. 60°
6. 65°
7. 60°
8. 135°
9. 90°
10. 75°

Work Unit 5-2

1. Right angle
2. Obtuse angle
3. Acute angle

Work Unit 5-3

1. $18^\circ\ 4'$
   $+180^\circ$
   $198^\circ\ 4'$

2. $38^\circ\ 48'$
   $+180^\circ$
   $218^\circ\ 48'$

3. $118^\circ\ 29'\ 29''$
   $+140^\circ\ 42'\ 24''$
   $259^\circ\ 11'\ 53''$

4. $60^\circ\ 14'\ 2''$
   $+18^\circ\ 27'\ 17''$
   $78^\circ\ 41'\ 19''$

5. $210^\circ\ 7'$
   -180°
   $30^\circ\ 7'$

6. $274^\circ\ 34'$
   -180°
   $94^\circ\ 34'$

7. $124^\circ\ 17'\ 35''$
   $-82^\circ\ 18'\ 34''$
   $41^\circ\ 59'\ 1''$

8. $25^\circ\ 16'\ 7''$
   $-14^\circ\ \ \ 8''$
   $11^\circ\ 15'\ 59''$

Work Unit 5-4

1. $A = s^2$

2. $A = s^2$
   $A = (48)^2$
   $A = 2,304$ sq in.

3. $A = 5^2$
$A = (16.5)^2$
$A = 272.25$ sq in.

Work Unit 5-5

1. $A = lw$

2. $A = lw$
$A = (18)(12)$
$A = 216$ sq in.

3. $A = lw$
$A = (22.5)(10.5)$
$A = 236.25$ sq in.

Work unit 5-6

1. $A = bh$

2. $A = bh$
$A = (24)(12)$
$A = 288$ sq in.

3. $A = bh$
$A = (50.2)(15.3)$
$A = 768.06$ sq in.

Work Unit 5-7

1. $A = 1/2h(b_1 + b_2)$

2. $A = 1/2h(b_1 + b_2)$
$A = 1/2(10)(28 + 34)$
$A = 5(62)$
$A = 310$ sq in.

3. $A = 1/2h(b_1 + b_2)$
$A = 1/2(12)(15.5 + 23.4)$
$A = 6(38.9)$
$A = 233.4$ sq in.

Work Unit 5-8

1. $A = 1/2bh$

2. $A = 1/2bh$
$A = 1/2 \times 20 \times 15$
$A = 150$ sq in.

3. $A = 1/2bh$
$A = 1/2 \times 22.4 \times 16.5$
$A = 184.8$ sq in.

Work Unit 5-9

1. $C = \pi d$

2. $C = 2\pi r$
$C = 2(3.14)\ 7.5$
$C = 47.1$ in.

3. $C = \pi d$
$C = 3.14\ (19)$
$C = 59.66$ in.

Work Unit 5-10

1. $A = \pi r^2$

2. $A = \pi r^2$
$A = 3.14\ (3.5)^2$
$A = 3.14\ (12.25)$
$A = 38.465$ sq in.

3. $A = \pi r^2$
$A = 3.14\ (11^2)$
$A = 3.14\ (121)$
$A = 379.94$ sq in.

Work Unit 5-11

1. $V = e^3$

2. $V = e^3$
$V = 7^3$
$V = 343$ cu. in.

3. $V = e^3$
$V = 16^3$
$V = 4,096$ cu. in.

Work Unit 5-12

1. $V = lwh$

2. $V = lwh$
$V = 34 \cdot 17 \cdot 16$
$V = 9,248$ cu. in.
$\frac{9,248}{1,728} = 5.35$ cu. ft.

3. $V = lwh$
$V = 21 \cdot 12 \cdot 5$
$V = 1,260$ cu. in.

Work Unit 5-13

1. $V = \pi r^2 h$

2. $V = \pi r^2 h$
$V = 3.14 \times 9.5^2 \times 40$
$V = 3.14 \times 90.25 \times 40$
$V = 11,335.4$ cu. in.

3. $V = \pi r^2 h$
$V = 3.14 \times 7.5^2 \times 32$
$V = 3.14 \times 56.25 \times 32$
$V = 5,652$ cu. in.

Work Unit 5-14

1. $\sqrt{1038.0000}$ = 32.21

	9
~~60~~	138
62	124
~~840~~	1400
642	1284
~~6440~~	11600
6441	6441
	5159

2. $\sqrt{511225.}$ = 715

	49
~~140~~	212
141	141
~~1420~~	7125
1425	7125

3. $\sqrt{33.0000}$ = 5.74

	25
~~100~~	8 00
107	7 49
~~1140~~	5100
1144	4576
	524

4. $\sqrt{94.8676}$ = 9.74

	81
~~180~~	1386
187	1309
~~1940~~	7776
1944	7776

Work Unit 5-15

1. $a^2 + b^2 = c^2$
$5^2 + 6^2 = c^2$
$25 + 36 = c^2$
$61 = c^2$
$\sqrt{61} = c$
$7.81 = c$

$\sqrt{61.0000}$ = 7.81

	49
~~140~~	1200
148	1184
~~1560~~	1600
1561	1561
	39

2. $a^2 + b^2 = c^2$
$3^2 + 9^2 = c^2$
$9 + 81 = c^2$
$90 = c^2$
$\sqrt{90} = c$
$9.48 = c$

$\sqrt{90.0000}$ = 9.48

	81
~~180~~	9 00
184	7 36
~~1896~~	1 6400
1888	1 5104
	1296

3. $a^2 + b^2 = c^2$
$24^2 + b^2 = 26^2$
$576 + b^2 = 676$
$b^2 = 100$
$b = \sqrt{100}$
$b = 10$

$\sqrt{100.}$ = 10

	1
20	000

4. $a^2 + b^2 = c^2$
$a^2 + 15^2 = 39^2$
$a^2 + 225 = 1521$
$a^2 = 1296$
$a = \sqrt{1296}$
$a = 36$

$\sqrt{1296.}$ = 36

	9
~~60~~	396
66	396

Work Unit 5-16.

1. $\frac{14}{17} = \frac{28}{x}$

$14x = 476$

$\frac{14x}{14} = \frac{476}{14}$

$x = 34$

2. $\frac{16}{14} = \frac{x}{27}$

$14x = 432$

$\frac{14x}{14} = \frac{432}{14}$

$x = 30.85$

STUDY UNIT 6

PROBLEM SOLVING

STUDY UNIT OBJECTIVE: UPON SUCCESSFUL COMPLETION OF THIS STUDY UNIT, YOU WILL BE ABLE TO LIST THE STEPS OF A PLAN FOR SOLVING WORD PROBLEMS, AND BE ABLE TO APPLY THIS PLAN TO PROBLEMS INVOLVING FORMULAS AND ALSO TO GENERAL-TYPE WORD PROBLEMS.

In a recent worldwide study of junior high school math students, the U.S. placed a poor eleventh place in grades on an achievement test. Many factors were attributed to being the cause. One of the important ones was the innate "fear" of mathematics exhibited by the American students. The topic of this study unit probably evokes more of this fear than any other math on the secondary school level. Just mention to a student that he is going to do "word problems" and he clutches up, gets a glassy stare in his eyes, and says, "Man, I can't do word problems." If however, you were to ask this same person a mathematical question and not have him read it, that is, give him the question in a situation completely divorced from the usual word problem unit in the course, odds are that he would use some logic and common sense and solve it. After all, isn't this the way that most problems are in real life, that is, won't the math problem have a situation involved with it? Why then don't students apply some of this logic to the formal classroom unit on word problems? It is usually only this written form of the problem that alarms the student. If word problems were attacked with a plan just as we attack an objective by first organizing everything into a logical sequence called a 5-paragraph order, much of the fear would be replaced with confidence in one's ability to solve problems. Each topic that has been discussed in this course can have word problems to accompany it. The exercises in percentage, proportion, and working with triangles, for example, had simple word problems associated with them. Recall from the example of the Rhind Papyrus, that word problems are almost as old as mathematics itself. Many of the typical problems involving age, money, or number relations have no practical value, but simply help to give an appreciation of number values and relationships. In most of the more modern textbooks, there are specific types of word problems stressed, again many of which have no practical application to most people, but ones that could come from real situations. The type of problems most stressed now are geometric formulas, distance, work, and mixture items. The most practical application for the Marine Corps would probably be the problems involving geometric forms. In this study unit we will present a plan for solving problems, examples of several types of problems that you may encounter, and some exercises to let you practice.

Work Unit 6-1. THE PLAN

DETERMINE THAT ALL WORD PROBLEMS SHOULD BE SOLVED BY APPLYING A LOGICAL PLAN.

Just about any successful endeavor has a sound plan to go with it. Build an airfield, write a book, wire a camp, clear a village, lay a minefield, teach a lesson, win a game, take a hill, or solve a word problem, a plan will aid in the completion of the project. The plan for solving problems may involve four, five, or six steps depending on how the operations are combined. Keep in mind that many problems can be solved by trial and error with arithmetic, much the same as the ancient Egyptians did them. In most problems this trial and error aspect can be eliminated by applying a simple equation or formula to the information that is known. Hence this will be an integral part of our plan. From this, you should not infer that all problems should be solved by algebra. The method of solution for some questions will be obvious to you on first reading, with just an arithmetic operation necessary for solution. The application of this plan, however, can save time. Our plan will consist of five steps:

Read the problem. This is self-explanatory. It would seem obvious, but many mistakes are made because the student skips over some important aspect. Where the problem is not written, but is some real-life situation, write all of the facts down so that the complete thing can be analyzed. Don't read into the problem things that are not there.

Determine the unknowns and represent them. You will have certain information given to you about a problem. There is usually a primary question asked. You must determine the method of going from this information to the solution. If the problem involves a formula or equation, there are unknown items that must be represented. Use any device that you can think of to aid you in this. Common aids that are employed are diagrams and charts. Use of these will be seen in the examples.

Write an equation. If the problem involves an equation, use the information from the preceding step to write one. If it involves a formula, substitute the known values for the appropriate letters of the formula. If it can be solved with arithmetic, decide which operations are needed.

Solve the equation. Solve and check the formula or equation or perform the arithmetic. Do not neglect the check.

Answer the question. The answer to an equation, a formula, or an arithmetic operation may not be the answer that you are looking for. Be sure to express your answer in the terms of the original problem. For example, you may want to know square feet, but your equation has given you an answer in square inches. Another arithmetic operation would be necessary to obtain this.

EXERCISE: Answer the following items and check your answers in the back of the study unit.

1. All word problems should be solved by applying a logical _____

2. List the five steps used in applying a logical plan to word problems.

   (1)
   (2)
   (3)
   (4)
   (5)

Work Unit 6-2. THE LANGUAGE

GIVEN AN ENGLISH EXPRESSION, TRANSLATE INTO MATHEMATICAL SYMBOLS.

In solving a problem, your initial job will be to translate English phrases into the language of mathematics. This is a matter of associating words with mathematical symbols. For example, the operation of addition symbolized by the plus sign can take the place of: more than, exceeds, increased by, and the sum of. Subtraction or the minus sign replaces; the difference between, less than, decreased by, and diminished by. Multiplication is indicated by times, of, and the product of; and division by the quotient of. Often a combination of operation signs and signs of grouping are used. For example, the statement six more than the difference of five and two could be written (5-2) + 6. Although the use of parentheses makes it clearer, it actually is not needed because of the order of operations. Here are some other examples of translating words to mathematical symbols:

The sum of six and 19	$6 + 19$
Eight increased by 12	$8 + 12$
The difference between 17 and 2	$17 - 2$
The product of two and nine	2×9
x diminished by twenty	$x - 20$
The quotient of m divided by seven	$\frac{m}{7}$
The sum of a and b, divided by 4	$\frac{a + b}{4}$

1 less than n	$n - 1$
x less than y	$y - x$
One-third of the product of x and y	$\frac{xy}{3}$
One divided by twice the sum of a and b	$\frac{1}{2(a + b)}$
The product of 17 and 3 divided by 2 times their difference	$\frac{17 \times 3}{2(17 - 3)}$

Note that we are not asking for solutions to the statement, just translations. As with most of mathematics, practice is required in order for you to understand the translations.

Before getting into the solution of problems, let us look at some problem situations that involve use of the language. As above, we will just be translating into symbols. Diagrams will be used to help depict some of the situations.

Examples:

An airfield has two runways. One is twice as long as the other. Draw lines to represent the runways and label each in terms of the same variable.

This could be done two ways. First, draw the lines.

Now, to label them we must select some convenient representation for either line. Let us say that we call the smaller runway x distance in length.

If the shorter one is x and the longer one is twice as long, then it would be 2x.

Suppose that we had labeled the longer one as x

If this is twice as long as the shorter one, then that means that the shorter one is half as long: $\frac{1}{2^x}$ or $\frac{x}{2}$

Many problems will have several ways in which they can be expressed. Be aware of this.

The floor of a building is made of 90 boards laid side by side. If w equals the number of inches in the width of one board, express the width of the room.

If one board is w, then 90 boards must be 90 times w or 90w.

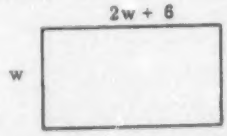
A company produced h pairs of jungle boots during its first month. Production increased by 5000 pairs each succeeding month. Express the number of pairs produced during the third month.

This can be shown quite handily in chart form.

1st month	2d month	3d month
h	h + 5000	(h + 5000) + 5000

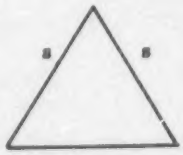
A hut is 6 feet longer than twice its width. Represent the number of feet in the width and the length.

The figure is a rectangle. We can start by letting w equal the width. The length then is 6 added to twice this width, or 2w + 6



The base of a triangle with two equal sides is 4 feet less than the sum of the two equal sides. Represent the length of each side, the base, and the perimeter.

Draw the figure. By the way, what kind of triangle is it? Did you say isosceles? You are correct. Label these two equal sides with the same letter.



The base is 4 feet less than the sum of these two sides.

(s + s)-4
or
2s-4

The perimeter would be the sum of the three sides:

s + s + 2s - 4
or
4s - 4

Now it is your turn:

EXERCISES: Translate the English expressions into mathematical symbols. Check your answers in the back of the study unit.

1. The difference between 2x and five.
2. c added to d
3. Fifteen decreased by n
4. The product of 6 and a
5. Four more than 2 times eight
6. The difference between twice x and half of y
7. Six times the sum of n and 3
8. Twenty-seven reduced by 5 times d
9. 5 divided by the sum of 4 times x and 3 times n
10. One-half of the sum of r and s.
11. There are twelve rows of seats in the bleachers. Each row is n inches above the one in front. How high is the top row?
12. One number is $\frac{4}{5}$ of another number. If the larger number is represented by m, write an equation stating that the sum of the two numbers is 108.
13. A company produced 400 M-117654's during its second year of operation. If this is an increase of x over the first year, express the production during the first year.
14. At the PX, a notebook costs 28 cents more than a pencil. Represent the total cost of 5 pencils, and 2 notebooks. Let x represent the cost of one pencil.

Work Unit 6-3. THE PROBLEMS

GIVEN WORD PROBLEMS, APPLY A LOGICAL PLAN AND SOLVE.

There are seven houses; in each are seven cats. Each cat kills seven mice. Each mouse would have eaten seven ears of spelt (wheat). Each ear of spelt will produce seven hekats of grain. What is the total of these. (That is, how much grain was saved?)

As I was going to St. Ives, I met a man with seven wives. Each wife had seven cats. Each cat had seven kits. Kits, cats, husband, and wives, how many were going to St. Ives?

The first problem which is from the Rhind Papyrus, and the second which is an ancient variation of it, are clearly meant for amusement. By reading them carefully, you could surely solve them both and probably not be the least concerned that they are word problems. In this paragraph we will not be solving any other puzzles, but will confine ourselves to problems with a practical value. Read them through as carefully as you would read a puzzle-type problem and apply some of the ideas presented so far in this study unit. Oh! by the way, the answer to the first puzzle is 16,807 hekats, and the answer to the second problem is one person going to St. Ives.

Problems involving formulas. These are by far the easiest type of word problems since the formula is the equation and it is only necessary to plug the pertinent items into it. The type of formula most often encountered will probably involve some aspect of geometric figures. Conversely, keep in mind that whenever the problem discusses a geometric figure that there is a formula to aid you. Always sketch the figure, label the parts with the information that you have, and then apply them to the proper formula.

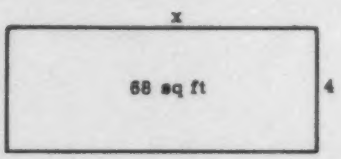
Example 1:

The area of a rectangle that is 4 feet wide is 68 square feet. Find the length.

This should prove no problem to you since it is similar to what you have already done in study unit 5. Let us use the five steps to solve the problem.

(1) Read the problem.

(2) Determine unknowns and represent them. Here a diagram does this.



(3) Write an equation.

$A = lw$
$68 = l(4)$

(4) Solve the equation.

$A = lw$

$68 = l(4)$
$68 = 4(l)$
$17 = l$

Check $68 = 17(4)$

$68 = 68$

(5) Answer the problem.

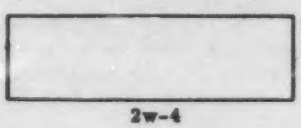
The length of the rectangle is 17 feet.

Example 2:

The length of a picture is 4 feet less than twice its width. To frame it, 76 feet of framing are needed. Find the dimensions of the picture.

(1) Read the problem.

(2) Determine unknowns and represent them. Again a diagram does this.



width = w
length = 2w-4
perimeter = 76

(3) Write an equation.

$P = 2l + 2w$
$76 = 2(2w-4) + 2(w)$

(4) Solve the equation.

$P = 2l + 2w$
$76 = 2(2w - 4) + 2(w)$
$76 = 4w - 8 + 2w$
$76 = 6w - 8$
$84 = 6w$
$14 = w$

If you are not sure of the multiplication of $2(2w-4)$, review the distributive law.

Check

$76 = 2(2w-4) + 2(w)$
$76 = 2(2 \times 14 - 4) + 2(14)$
$76 = 2(28 - 4) + 28$
$76 = 2(24) + 28$
$76 = 76$ ✓

(5) Answer the question:

The dimensions are: width = 14 ft, length equal 2w-4 or 24 feet. This can also be checked by substituting these values in the formula.

$P = 2l + 2w$
$76 = 2(24) + 2(14)$
$76 = 48 + 28$
$76 = 76$

Example 3:

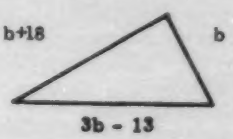
One side of a triangular lot is 18 feet more than the second. The third side is 13 feet less than 3 times the second. To fence the lot requires 130 feet of fencing. Find the length of each side of the lot.

(1) Read the problem.

(2) Represent the unknowns and draw a diagram. You should have noticed in reading the problem that the first and third sides are expressed as they relate to the second side. The second side then should be the first unknown that you represent.

Let b = 2d side

Now with this single letter variable as a start, you should be able to translate the language of the problem into symbols.
Let $b + 18$ = 1st side
Let $3b - 13$ = 3d side



(3) Write an equation. The formula is: $P = a + b + c$

$$130 = (b + 18) + b + (3b - 13)$$

Parentheses are not needed, but are inserted to show the three different sides.

(4) Solve the equation.

$130 = (b + 18) + b + (3b - 13)$
$130 = b + 18 + b + 3b - 13$
$130 = 5b + 5$
$125 = 5b$
$25 = b$

Check: $130 = (b + 18) + b + (3b - 13)$
$130 = (25 + 18) + 25 + (3 \cdot 25 - 13)$
$130 = 43 + 25 + 75 - 13$
$130 = 130$

(5) Answer the question.

To do this, we must go back to the unknown representations.

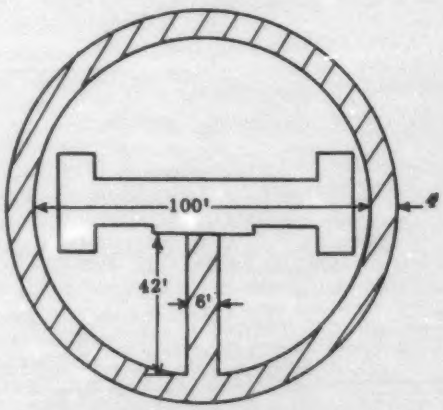
$b = 25$
$b + 18 = 43$
$3b - 13 = 62$

Check: $a + b + c = 130$
$43 + 25 + 62 = 130$
$130 = 130$

This time let's take a problem that is a little more complicated (not more difficult, just more steps involved) and again apply the five steps. See if you can anticipate the steps of the solution before they are presented.

Example 4:

The division headquarters building is to have a circular sidewalk placed around it. The walk will be 4 feet wide and 4 inches thick. There will also be a 6-foot-wide sidewalk from this walk to the entrance. Sgt Readymix, the combat engineer chief, has his men ready to go to work as soon as the concrete arrives. He has ordered 19 1/2 yards of concrete. Doublecheck Sgt Readymix and see if he has ordered the correct number of cubic yards of concrete needed. Consult the illustration for all dimensions.



(1) Read the problem.

(2) Determine the unknowns and represent them. Here, all of the needed information is given in the diagram. The only real unknown is the answer. The biggest problem for you now is to select the formula or formulas that are to be used. What geometric forms are involved? Clearly there are two: one circular, one rectangular.

Since the sidewalk is to be 4 in. thick, keep in mind that each figure is three-dimensional. As usual with most problems, several methods can be used to solve this problem. One method will be presented, but it is not necessarily THE method. No matter the method, the key to this problem is finding the area of the shaded strip of the circle, that is, the area of the sidewalk. What formula would you use to solve the problem?

(3) Write an equation. The equations will be the formulas. To find the area of the circular sidewalk involves the formula $A = \pi r^2$. First we must use it with the information from the large circle and then with the information from the shaded circle illustrated here. If the large area is found and then the area of the shaded circle subtracted from it, the remainder will be the area of the sidewalk.

Large		Small
$A = \pi r^2$		$A = \pi r^2$
$A = 3.14\ (54^2)$	minus	$A = 3.14(50^2)$

Again keep in mind that the 4 in. thickness must be considered and that it must be changed to feet. The difference of the two areas multiplied by 1/3 ft will give the volume in cubic feet of the circular walk. The short straight sidewalk is a rectangular solid for which we will use:

$$V = lwh$$
$$V = 42 \times 6 \times 1/3$$

(4) Solve the equations.

Large circle	Small circle	Rectangular solid
$A = \pi r^2$	$A = \pi r^2$	$V = lwh$
$A = 3.14\ (54^2)$	$A = 3.14(50^2)$	$V = 42 \times 6 \times 1/3$
$A = 3.14\ (2916)$	$A = 3.14(2500)$	$V = 84$ cubic feet
$A = 9156.24$	$A = 7850$	

(5) Answer the problem. In this case, the solutions to the formulas are not the answers to the problem. As indicated above, we must find the difference between the two circle areas:

Difference:

$$\begin{array}{r} 9156.24 \text{ sq ft} \\ -\ 7850.00 \text{ sq ft} \\ \hline 1306.24 \text{ sq ft} \end{array}$$

Multiply this by 1/3 ft

$1306.24 \times 1/3 = 435.41$ cubic feet

Add the two volumes

$$\begin{array}{r} 435.41 \text{ cubic feet - circular sidewalk} \\ +84.00 \text{ cubic feet - straight sidewalk} \\ \hline 519.41 \text{ cubic feet concrete needed} \end{array}$$

Is this the answer?

No. The problem asked how many cubic yards of concrete were needed. Since there are 27 cubic feet in 1 cubic yard (3 x 3 x 3), if 519.41 is divided by 27, the answer will be the solution to the problem.

$$\frac{519.41}{27} = 19.23 \text{ cubic yards}$$

Checking back to the original problem Sgt Readymix ordered 19 1/2 yards. This is correct for the job.

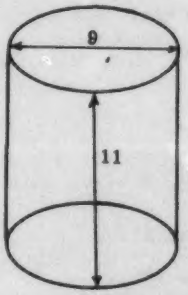
Let us have one more detailed example before letting you get some practice.

Example 5:

Water is stored in a cylindrical tank with a height of 11 meters and a diameter of 9 meters. What is the capacity, in gallons, of the tank if there are approximately 7 1/2 gallons in 1 cubic foot of water? If the tank is hooked to a shower unit and each man is allowed 5 gallons of water, how many men would it take to empty the tank?

As you can see, the problem involves several steps, and meters are inserted just to prod the thought processes a little. The problem can be solved in several ways, and again one method will be shown.

After thoroughly reading and analyzing the problem, it is determined that the only formula needed is the volume of a cylinder, $V = \pi r^2 h$. Before using the formula, however, we will change the meters to feet since we must eventually know cubic feet in order to determine gallons. One meter equals approximately 3.281 feet. Therefore:



9 meters = 9 x 3.281 = 29.529 feet
11 meters = 11 x 3.281 = 36.091 feet

Round off to two places, 29.53 and 36.09.

Now we can substitute the approximate values in the formula and solve the equation.

Do the substitution step and then check below to see if you have it correct.

$V = \pi r^2 h$
$V = 3.14\ (14.77^2)\ 36.09$

Did you get it correct or did you forget to take half of the diameter? If you got it right, congratulate yourself. This is a common error.

$V = \pi r^2 h$
$V = 3.14(14.77^2)\ 36.09$
$V = 3.14(218.153)\ 36.09$
$V = 24,721.65$ cubic feet

This is the capacity, in cubic feet, of the tank.

Now if every cubic foot contains 7 1/2 gallons, that means that the product of 24,721.65 and 7 1/2 will be the gallon capacity of the tank.

$24,721.65 \times 7\ 1/2 = \underline{185,412.38 \text{ gallons}}$

If each man is allowed 5 gallons of water, then dividing the capacity of the tank by 5 will tell us how many men can take a shower.

$$\frac{185,412.38}{5} = 37,082.47$$

Therefore 37,083 men will empty the tank. Take your time and have a good shower.

Miscellaneous word problems. Depending on your occupation, you may never actually have the opportunity or occasion to solve word problems other than those involving a formula or one simple arithmetic operation. However, being able to think and rationalize with numbers instills confidence in your own abilities and helps you to develop the logical and analytical thinking needed in many Marine Corps endeavors. To look at a problem, analyze it, and come up with an answer or decision is a desirable ability. To this end, the random problems in this paragraph may or may not apply to you and your job. The key thing is that they stimulate you to think and allow you to carry away principles that can be transferred to similar problems in other situations. In some instances, complete problems will not be presented, but you will be asked again only to translate English to mathematical symbols. As in other parts of the course, several example problems will be worked and then an exercise will be presented for you to test your skill. Keep in mind that even if an equation is not required to solve the problem, you should still follow a plan. Read the problem thoroughly, decide what is known and what is to be found, perform the arithmetic operations and check them, and answer the problem in the terms asked for in the question. Also apply some commonsense to decide whether your answers seem logical.

Example 1:

Shore party has 60 men working in 6 dumps on the beach. If no dump has less than 7 men and no more than 18, what is the minimum number of Marines that can work in any two of these dumps?

This problem does not have a practical value, it is just a number exercise. At first glance the problem may seem complicated, but it is really quite basic. No equation is needed; just read it carefully. It asks the minimum number of Marines that can work at two of the dumps. If 7 is the minimum for one, than $7 \times 2 = 14$ must be the minimum for two. Just to be certain, however, we must check to be sure that this will not make any of the other dumps exceed 18. From the total of 60, subtract 14 which leaves 46 for the four other dumps. This is within the limits set, so our answer is OK.

Example 2:

There are 40 questions on a promotion test. Each correct answer is worth one point and two points are deducted for each wrong answer. No points are added or deducted for questions not answered. If LCpl Claymore gets a score of 15 and he had 5 questions wrong, what fractional part of the test did he answer? Read the question thoroughly.

There are several ways to get the answer. The problem will fit very nicely into an equation. From the information given, we know that the score is equal to the number correct minus two times the number incorrect.

If C = correct
I = incorrect
$C - 2I = 15$

We know that there were 5 incorrect problems so

$$C - 2(5) = 15$$
$$C - 10 = 15$$
$$C = 25$$

Check: $25 - 10 = 25$
$15 = 15$

Claymore got 25 correct and 5 incorrect. This means he answered 30 of the 40 problems, or

$$\frac{30}{40} = \frac{3}{4}.$$

Example 3:

A local discount store has vitamins on sale for 21 cents a dozen. The PX has the same item 100 for $1.50. How much is saved per dozen by buying the larger amount?

As usual the problem can be solved in several ways. One will be presented.

After reading the problem, it can be determined that algebra is not needed, just two simple arithmetic operations.

First, the problem asks for a comparison of dozens. One price is already in dozens. The other can be converted several ways. Since 100 vitamins cost $1.50, and you remember from earlier in the course about dividing by pwers of ten, you know that each vitamin costs 1.5 cents. If one costs 1.5 cents, then 12 will cost 1.5 x 12 = 18 cents. The difference between the two prices is .21 - .18 = 3 cents per dozen.

This is a basic type of problem, but a practical one that appears often. Keep it in mind, it can save you money.

Example 4:

The indicator of a fuel tank shows 1/5 full. After the tanker delivers 165 gallons of fuel, the indicator shows 4/5 full. What is the capacity of the tank?

The first thing to observe is that if the tank was 1/5 full at the start and 4/5 full after the fuel was delivered, then the amount delivered was 4/5 - 1/5 = 3/5 of the total capacity. We also know that this 3/5 is the same as 165 gallons. From earlier work in percentage, you should remember how to write a sentence with the facts involved. In place of a percent we will have a fraction.

If the capacity of the tank is x, then:

165 gallons is $\frac{3}{5}$ of x.

$165 = \frac{3}{5}x$

$\frac{5}{3}$ $165 = x$

$275 = x$

Check: $165 = \frac{3}{5}(275)$

$165 = 165$

The capacity of the tank is 275 gallons.

Obviously in four example problems we have not given the solution to all of the problem-solving situations that exist. It is hoped, though, that if you worked the problems as they were presented and followed the explanations, that you have more of a "feel" for problem solving and will not view it with alarm. Again, remember to read the problem thoroughly and follow a plan.

EXERCISE: Solve the following problems and check your answer in the back of the study unit.

1. Several Marines pooled their resources and bought a used car. One of them owned $\frac{5}{8}$ interest in it. He sold $\frac{1}{5}$ of his interest to another Marine for $100. What was the original price of the car?

2. In 9 weeks, a moonlighting Marine working 5 nights a week made $454.50. How much did he make per day?

3. A company plans to raise $500 for the Navy Relief drive by selling chances on a motorcycle . If expenses are $250, what is the minimum number of tickets at 75 cents each that must be sold to reach the goal?

4. After several days of working out, 20% of the canidates for a football team were told to turn in their jocks. This left 32 players on the team. How many tried out originally.

5. A badminton court 44 feet long is 4 feet longer than twice its width. Find the width.

6. The troop space in an LVT measures 5 ft 6 in. high, 7 ft 3 in. wide, and 11 ft 5 in. long. If boat space for one Marine is considered to be 13.5 cu ft, how many Marines, can be jammed into an LVT? If there are 5 tractors available, could a company of 195 men be carried in one wave?

SUMMARY REVIEW

In the preceding work units you have been introduced to word problems. You have determined that all word problems should be solved by applying a logical plan. You can now translate english expressions into mathematical symbols. When given word problems, you can apply a logical plan and solve them.

Answers to study unit #6 Exercises

Work Unit 6-1.

1. plan
2. (1) Read the problem.
   (2) Determine the unknowns and represent them.
   (3) Write an equation.
   (4) Solve the equation.
   (5) Answer the question.

Work Unit 6-2.

1. $2x - 5$
2. $c + d$
3. $15 - n$
4. $6a$
5. $2 \times 8 + 4$
6. $2x - \frac{y}{2}$
7. $6(n + 3)$
8. $27 - 5d$
9. $\frac{5}{4x + 3n}$
10. $\frac{r + s}{2}$
11. $12n$
12. $\frac{4}{5}m + m = 108$
13. $400 - x$
14. $5x + 2(x + 28)$

Work Unit 6-3.

1. $800.00

$1/5 = 20\%$

$\frac{\$100}{x} = \frac{20\%}{100\%}$ (Let x represent investment for 5/8 of car).

$\frac{100}{x} = \frac{.20}{1.00}$

$.20x = 100$

$\frac{.20x}{.20} = \frac{100}{.20}$

$x = \$500.00$ (Total investment for 5/8 of car).

$\frac{5}{8} = 62.5\%$

$\frac{\$500}{x} = \frac{62.5\%}{100\%}$ (Let x represent the original price of car).

$\frac{500}{x} = \frac{.625}{1.00}$

$.625x = 500$

$\frac{.625x}{.625} = \frac{500}{.625}$

$x = \$800.00$ original price of car

2. $10.10
5 nights a week for 9 weeks = 45 nights
$454.50 ÷ 45 = $10.10 per night

3. 1,000 tickets
$500 + $250 = 750
750 ÷ .75 = 1,000 tickets must be sold

4. 40 players tried out originally.

$\frac{32}{x} = \frac{80\%}{100\%}$ (Let x represent total amount of players that tried out originally).

$\frac{32}{x} = \frac{.80}{1.00}$

$.8x = 32$

$\frac{.8x}{.8} = \frac{32}{.8}$

x = 40 players tried out originally

5. 20' wide

$\frac{44' - 4'}{2}$ = 20' wide

6. 33.7 or 34. No. Five tractors could only carry 170 men.

5'6" = 66"
7'3" = 87"
11'5" = 137"

Formula for volume of a rectangle is l x w x h

137"
x 87"
11.919"

11,919"
x 66"
786,654

$$1728 \overline{)786,654"} \quad 455.23958 \text{ cu. ft.}$$

1728 cu in. = 1 cu ft.

$$13.5. \overline{)455.2.4} \quad 33.72$$

33.7 or 34 men in one LVT

34
x 5
170 men can be carried in five LVT's

MATH FOR MARINES

Review Lesson

Instructions: This review lesson is designed to aid you in preparing for your final exam. You should try to complete this lesson without referring to the course text or other materials, but if you find that you must use the materials to answer some of the questions, do so. The enclosed answer sheet must be filled out according to the instructions on the back of the sheet and mailed to MCI using the envelope provided. If you answer any items incorrectly, they will be listed with the appropriate reference on a feedback sheet which will be mailed to your commanding officer with your final exam. You should study the referenced material for the questions you missed before taking the final exam. Use the protractor provided with the course to answer items 66 and 67.

A. Multiple Choice: Select the ONE answer which BEST completes the statement or answers the question. After the corresponding number on the answer sheet, blacken the appropriate circle.

Value: 1 point each.

1. The Hindu-Arabic number system was superior to all others because of the

   a. position of the numbers.
   b. symbols used for each number.
   c. invention of zero.
   d. tally marks used in the system.

2. One purpose of the digits in any number is to show the place value and the other purpose is to show the _____ value.

   a. model
   b. power
   c. natural
   d. cardinal

3. Which set is <u>not</u> made up entirely of natural numbers?

   a. -2, 0, 4
   b. 4, 6, 8
   c. 3, 6, 9
   d. 10, 11, 12

4. Which item does <u>not</u> illustrate the principle of closure?

   a. 6 x 6=
   b. 3/4 x 5=
   c. 18 + 20=
   d. 37 +17=

5. A system of whole numbers made up of both natural and negative numbers is known as

   a. product.
   b. irrational numbers.
   c. integers.
   d. sum.

6. The rational numbers are a system of numbers made up of natural numbers, integers, and

   a. fractions.
   b. irrational numbers.
   c. groups.
   d. expressions.

7. The three axioms of equality are the transitive, symmetric, and _____ properties.

   a. commutative
   b. reflexive
   c. distributive
   d. associative

8. The addition 16 + 25 and the multiplication 33 x 46 can be written 25 + 16 and 46 x 33 because of the _____ property.

a. reflexive
b. unity
c. commutative
d. distributive

9. The addition (64 + 72) + 85 and the multiplication (64 x 72) x 85 can be written 85 + (64 + 72) and 64 x (85 x 72) because of the _____ law.

a. transitive
b. distributive
c. associative
d. commutative

10. The distributive property states that

a. addition can be distributed over multiplication.
b. multiplication can be distributed over addition.
c. addition can be distributed over division.
d. multiplication can be distributed over subtraction.

11. Solve the mathematical statement below.

$$15 \times 0 =$$

a. Impossible
b. 15
c. 1
d. 0

12. Solve the mathematical statement below.

$$1 \times 4$$

a. 1/4
b. 0
c. 4
d. Impossible

13. Simplify the mathematical statement below.

$$(24 \div 3) + 15 =$$

a. 87
b. 23
c. 21
d. 1

14. Simplify the mathematical statement below.

$$[20 + 2]\ 2 =$$

a. 44
b. 80
c. 42
d. 24

15. Simplify $[21 - 3 + 9 \div 3] \div 7 \times 7$

a. 1
b. 8.96
c. 21
d. 3/7

16. Simplify $(5 + 6 - 1) \div 2 + 1$

a. 6
b. 3 1/2
c. 21
d. 30

17. Reduce to lowest terms.

$$\frac{144}{264}$$

a. 3/7
b. 2/3
c. 12/16
d. 6/11

18. Reduce the fraction to higher terms by supplying the missing numerator.

$$\frac{6}{35} = \frac{?}{175}$$

a. 15
b. 29
c. 30
d. 5

19. Add:

$$\begin{array}{r} 8\ 4/9 \\ +\ 2\ 7/9 \\ \hline \end{array}$$

a. 11 2/9
b. 6 7/9
c. 11 11/9
d. 12 2/9

20. Subtract:

$$\begin{array}{r} 7\ 1/2 \\ -\ 6 \\ \hline \end{array}$$

a. 1/2
b. 1 1/2
c. 2
d. 6 1/2

21. Multiply:

$$9\ 1/2 \times 2\ 2/3$$

a. 25 1/3
b. 18 1/3
c. 18 1/6
d. 27 1/2

22. Divide:

$$3\ 2/3 \div 5/6$$

a. 5/22
b. 3 1/8
c. 23/5
d. 4 2/5

23. Change .36 to a common fraction and reduce to its lowest terms.

a. 9/50
b. 9/25
c. 18/100
d. 36/1000

24. Change 9/20 to a decimal fraction.

a. .0145
b. .045
c. .45
d. .920

25. Add:

18 + 7.0002 + 8.21 + 5.5

a. 38.7102
b. 300.7102
c. 38.712
d. 30.28

26. Subtract:

7.0002 - 1.5675

a. 8.5677
b. 6.5437
c. 5.0437
d. 5.4327

27. Multiply:

457 x 0.62

a. 2.0324
b. 28.334
c. 203.24
d. 283.34

28. Divide:

1.1 ÷ 6

a. 0.183
b. 1.8
c. 6.6
d. 66

29. Change .48 to a percent.

a. 4.8%
b. 48/100%
c. 48%
d. 4.8/100%

30. Change 4/9 to a percent.

a. 2.5%
b. 25%
c. 40%
d. 44.4%

31. Change 5/8% to a decimal.

a. 62.5
b. 6.25
c. .0625
d. .00625

32. Change 62% to a common fraction and reduce to lowest terms.

a. 31/50
b. 3/5
c. 3/10
d. 6/25

33. Of 155 people in the company, 80 need typhoid shots. What percent of the company need shots?

a. 1.9%
b. 12.4%
c. 51.6%
d. 124%

34. Of 155 people in the company, 60% went to boot camp at Parris Island. How many went to Paris Island?

a. 23
b. 25
c. 93
d. 103

35. 45% of the vehicles are deadlined. If there are 27 deadlined, what is the total number of vehicles?

a. 90
b. 54
c. 24
d. 12

36. $45x^3y$ is a(an)

a. polynomial term.
b. monomial term.
c. trinomial term.
d. binomial term.

37. Which pair of algebraic expressions below contains like terms?

a. $13ab^2, (5ab)^2$
b. $a^3, 8a^3$
c. $5x, 5x^3$
d. xy, xyz

38. Which one of the algebraic expressions below is a polynomial term?

a. $\frac{12x}{3x - y}$
b. $\frac{18xy}{x}$
c. $16xy$
d. $37a + 16a + a$

39. If $a = 2$ and $b = 5$, find the value of ab^3.

a. 30
b. 50
c. 250
d. 1000

40. Which statement best describes this algebraic sentence?

$$(8-5)^2 + 7^2 - 2^3 = \frac{(10-5)^2}{5} \cdot 2(4^2 - 11)$$

a. True equation
b. False equation
c. True inequality
d. False inequality

41. Which answer will correctly solve the equation and determine if it is true of false?

$$6 \cdot 3(9-2) = \frac{20-4}{8} + 12^2$$

a. 160 = 147, False
b. 198 = 204, False
c. 126 = 146, False
d. 27 = 27, True

42. Solve for x.

$8x + 10x + 10 = 28$

a. 1
b. 2.09
c. 18
d. 20.9

43. Solve for a.

$3a - 5\ 1/2 = 6\ 1/2$

a. 6
b. 4
c. 2
d. 1/2

44. Which answer will correctly solve the inequality and determine if it is true or false?

$$5(10-6) - \frac{9+3}{6} \neq 5 + 4 \cdot 3$$

a. $42 \neq 36$, True
b. $18 \neq 17$, True
c. $18 \neq 18$, False
d. $17 \neq 18$, True

45. Which answer will correctly solve the inequality and determine if it is true or false?

$$7 \cdot \frac{15+9}{6} < 3 \cdot 6$$

a. $28 < 18$, False
b. $18 < 24$, True
c. $19 < 18$, False
d. $24 < 18$, True

46. Find the value of x in the proportion.

$$\frac{5}{3x} = \frac{2}{12}$$

a. 0
b. 6
c. 8
d. 10

47. If three pints of a certain chemical are needed to purify 500 gallons of water, how many pints are needed to purify 800 gallons?

a. 8 2/5
b. 7
c. 4 4/5
d. 3 3/5

48. 71 is what % of 109?

a. 6.66
b. 15
c. 65.1
d. 150

49. 12 is 83% of what number?

a. 14.4
b. .144
c. 69
d. 6.9

50. Convert 1,634 meters to kilometers.

a. 1.634
b. 16.34
c. 163.4
d. 16,340

51. Convert 735 meters to centimeters.

a. 7.35
b. 73.5
c. 7,350
d. 73,500

52. On a 1:50,000 map, the distance between checkpoints Alpha and Bravo is 15 centimeters. What is the actual distance in meters?

a. 3,333
b. 7,500
c. 50,000
d. 75,000

53. Convert 3.5 hectoliters to liters.

a. .35
b. 35
c. 350
d. 3,500

54. Express the sum of 4.7 hectoliters and 137 milliliters in liters.

a. 4.7137
b. 47.137
c. 470.137
d. 4,701.37

55. Convert 1,750 grams to kilograms.

a. .175
b. 1.75
c. 17.5
d. 175

56. Express the sum of 13 metric tonne and 125,000 grams in kilograms.

a. 13.125
b. 131.25
c. 1,312.5
d. 13,125

57. Add:

6 yd, 2 ft
3 yd, 1 ft, 8 in.
1 ft, 10 1/2 in.
2 yd, 2 ft, 6 1/2 in.
2 ft, 7 in.
2 yd, 5 in.

a. 16 yd, 2 ft, 1 in.
b. 16 yd, 2 ft
c. 15 yd, 2 ft, 37 in.
d. 14 yd, 8 ft

58. Multiply:

4 gal, 10 qt, 5 pt, 18 oz
x 5

a. 56 gal, 10 oz
b. 36 gal, 1 qt, 10 oz
c. 36 gal, 30 pt, 10 oz
d. 26 gal, 1 qt, 10 oz

59. Add:

3 tons, 333 lb, 15 oz
9 tons, 201 lb, 4 oz
1,600 lb, 6 oz
+ 1 ton 12 oz

a. 13 tons, 936 lb
b. 13 tons, 1,134 lb, 37 oz
c. 14 tons, 136 lb, 5 oz
d. 16 tons, 37 oz

60. Convert 7,000 meters to yards.

a. 6,300
b. 7,777
c. 63,000
d. 77,770

61. 6 1/2 inches equal how many centimeters?

a. 1.625
b. 2.6
c. 16.25
d. 26

62. 45 liters equal how many quarts?

a. 4.275
b. 4.736
c. 42.75
d. 47.36

63. Change 55 gallons to liters.

a. 209
b. 144.7
c. 20.9
d. 14.47

64. Change 224 grams to ounces.

a. 5
b. 8
c. 16
d. 24

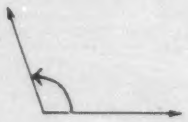
65. 180 pounds equals how many kilograms?

a. 81
b. 175
c. 265
d. 400

Note: Use the protractor provided with the course to find the measure of the angles in items 66 and 67.

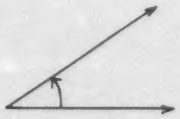
66. What is the degree of opening on the angle illustrated?

a. 30^{o}
b. 46^{o}
c. 95^{o}
d. 110^{o}



67. The angle illustrated below would be classified as an _____ angle.

a. right
b. obtuse
c. straight
d. acute



68. Add:

$$\begin{array}{r} 172^{o}\ 3' \\ +\ 8^{o}\ 57' \\ \hline \end{array}$$

a. 180^{o}
b. $180^{o}\ 60'$
c. 181^{o}
d. $190^{o}\ 54'$

69. Find the area of a square 13 1/2 inches on a side.

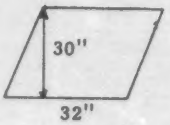
a. 202.35 sq in.
b. 182.25 sq in.
c. 108 sq in.
d. 54 sq in.

70. The length of a rectangle is 31.6 feet. If the width is 12 feet, what is the area?

a. 43.6 sq ft
b. 78.4 sq ft
c. 379.2 sq ft
d. 750 sq ft

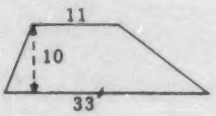
71. What is the area of the parallelogram illustrated?

a. 960 sq in.
b. 900 sq in.
c. 480 sq in.
d. 122 sq in.



72. What is the area of the trapezoid illustrated?

a. 150 sq units
b. 165 sq units
c. 176 sq units
d. 220 sq units



73. The base of a triangle is 36 inches. If the height is 24 inches, what is the area?

a. 60 sq in.
b. 324 sq in.
c. 432 sq in.
d. 864 sq in.

74. What is the circumference of a circle with a diameter of 28 inches?

a. 78.29 in.
b. 87.92 in.
c. 98.72 in.
d. 132.46 in.

75. Find the area of a circle with a radius of 14 inches.

a. 308 sq in.
b. 615.44 sq in.
c. 2464.23 sq in.
d. 4312 sq in.

76. Find the volume of a cube 9 inches on edge.

a. 81 cu in.
b. 416 cu in.
c. 729 cu in.
d. 834 cu in.

77. Find the volume of a foot locker 28 in. long by 18 in. wide by 16 in. high.

a. 7000 cu in.
b. 7064 cu in.
c. 8000 cu in.
d. 8064 cu in.

78. Find the volume of a cylinder with a radius of 10 inches and a height of 20 inches.

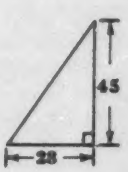
a. 628 cu in.
b. 1,837 cu in.
c. 4,840 cu in.
d. 6,280 cu in.

79. Find the square root of 484.

a. 22
b. 31
c. 47
d. 56

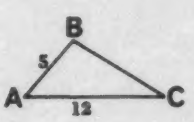
80. Find the missing dimension on the triangle illustrated.

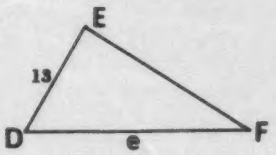
a. 49
b. 53
c. 60.01
d. 61.21



81. Triangle ABC is similar to triangle DEF. Find side e.

a. 31.2
b. 42.1
c. 60.2
d. 65





82. The best method used for solving word problems is to

a. set up equations.
b. apply simple arithmetic.
c. draw a picture.
d. apply a logical plan.

83. Express in mathematical symbols the sum of 6 and 8 increased by 1.

a. (6 + 8) - 1
b. 6 + (8 - 1)
c. (6 + 8) 1
d. 6 + 8 + 1

84. Express in mathematical symbols one-half the product of the base and the height.

a. 1/2 (b + h)
b. 1/2 bh
c. $\frac{b}{2} + h$
d. 1/2 $\frac{b}{h}$

85. Express in mathematical symbols eighteen reduced by 7 times y.

a. $\frac{18 - 7}{y}$
b. 18 + 7y
c. 18 - 7y
d. (18 - 7) y

86. A plane flew a triangular route a total distance of 149 miles. If the longest leg of the flight was twice the shortest leg, and the third leg was 1 mile less than than the longest leg, what was the length of the longest leg of the flight?

a. 30 miles.
b. 59 miles.
c. 60 miles.
d. 118 miles.

87. A cylindrical tank is 4 feet in diameter and 5 feet deep. If a canteen holds 1 quart, how many canteens can be filled from a full tank of water? (There are approximately 7 1/2 gallons in a cubic foot.)

a. 1884
b. 1500
c. 1024
d. 824

88. Several retired Marines pooled their money in a business venture. If Sgt Maj Oldsalt own 3/7 of the business and receives $6,000 as his share of the profit for the year, how much will MSgt Boot receive if he owns 2/7 of the business?

a. $1,614
b. $2,000
c. $2,562
d. $4,000

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Total Points: 88

* * *

STUDENT COURSE CONTENT ASSISTANCE REQUEST

COURSE NUMBER	COURSE TITLE	DATE:
NAME	RANK	MOS
SOCIAL SECURITY NUMBER		DATE:
COMPLETE MILITARY ADDRESS: (INCLUDE RUC)		
UNIT PHONE NUMBER		(PLEASE FILL IN ADDRESS ON REVERSE SIDE)

1. Use this form for any questions you may have about this course. Write out your question and refer to the study unit, work unit, or study question which you are having problems with. Complete the self-addressed block on the reverse side. Before mailing, fold the form and staple it so that MCI's address is showing. Additional sheets may be attached to this side of the form. Your question will be answered promptly by a Marine NCO!

MY QUESTION IS: _____ OUR ANSWER IS: _____

SSTD-1

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MCI - R14 K

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Section 1. Student Identification

Rank	Initials	Last Name	MOS

SSN ___ ___ ___ — ___ ___ — ___ ___ ___ ___ REPORTING UNIT CODE (RUC)

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INSTRUCTIONS: Print or type name, rank, and address clearly. Include ZIP CODE. Only Class III Reservists may use civilian address.

ZIP CODE

Section 2. CHECK THE APPROPRIATE BOX AND FILL IN THE APPROPRIATE SPACES.

FOR REGULAR AND CLASS II RESERVE MARINES THIS FORM MUST BE SIGNED BY THE COMMANDING OFFICER OR HIS REPRESENTATIVE, i.e. TRAINING NCO.

1. ☐ EXTENSION - Please grant an extension. (Will not be granted if already on extension.)

2. ☐ NOTICE OF COURSE COMPLETION - Final Exam Sent On _____. (New exam will be sent if exam not received at MCI.)

3. ☐ REENROLLMENT - Student has course materials (See para. 4003 of Vol. 1 of MCI Catalog for information on reenrollment.)

4. ☐ OVERDUE FINAL EXAM - Last (Review) lesson sent on _____. Please send exam.

5. ☐ Please send new ANSWER SHEETS.

6. ☐ Please send missing course materials (Not included in course package.)
Lessons_____ Manual_____ Other_____

7. ☐ CHANGE - Rank_____ Name _____
Social Security Number ☐☐☐-☐☐-☐☐☐☐
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8. ☐ OTHER (explain)_____

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